B-Spline Curve Smoothing Under Position Constraints for Line Generalisation

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ABSTRACT
Currently, most of the operations performed for the construction of marine charts are still done manually. However, with the development of more and more powerful techniques, new processing methods must be developed in order to deal with the increasing amount of data and to achieve automatic construction. For that purpose, a new method for line smoothing is introduced in this paper and is applied to the generalisation of isobathymetric lines (lines connecting points at a same depth). The lines are modelled by B-spline curves which maintain their smooth feature. Smoothing is performed by reducing the curvature using a snake model. The generalisation constraint of navigation safety is satisfied by applying position constraints on the line. Spatial conflicts are also taken into consideration and are removed during the process. Parameters are automatically defined so that the method can be applied to large sets without user intervention. The method has been applied on real data sets and examples are provided and discussed for scale reduction of lines.

Categories and Subject Descriptors
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1. INTRODUCTION
Marine charts can, according to the different purposes of the users, offer a schematic and simplified plane representation of the actual submarine relief. Thus, to construct a marine chart, data collected on the field must be selected, simplified or modified under definite conditions such as the scale of the map or its purpose. This stage of chart construction, called map generalisation, can be divided into two parts. The first part is object generalisation. It is mostly based on logical constraints and requires operations such as suppression, aggregation or grouping objects of a same class. Visual or aesthetic constraints are not taken into consideration. The second part is cartographic generalisation. In the process of converting the map to a smaller scale, cartographic generalisation reduces the complexity of a map, emphasises the prime information, and suppresses less relevant information. On the same time the links between different objects are maintained and the aesthetic quality of the map is preserved [20]. Objects are selected based on the scale of the map. Their geometric features are smoothed or emphasised with reference to their relevance. The main cartographic generalisation operations are aggregation, caricature, deletion, displacement, enlargement, smoothing.

In this paper, we discuss about line generalisation for marine chart construction. The lines are contour lines connecting the points at the same depth (isobathymetric lines). Data used for chart construction are usually obtained by sounding the seafloor and gathered in a bathymetric database. A Digital Elevation Model of the seafloor is then constructed and the isobaths are extracted from this DEM. In this paper, the only available data are the isobathymetric lines. As a consequence, generalisation cannot be performed by generalising the DEM [21]. Isobaths are usually defined with polygonal lines. Another suitable representation is the use of B-spline curves [17]. They are parametric curves defined by a polynomial function. Without the problem of broken arc effects as created by polygonal lines, B-spline curves can offer a smooth representation which improves compression and visualisation operations such as zooming or spanning.

Marine charts are subject to specific constraints that must be respected during the cartographic generalisation stage to ensure their legibility and navigation safety. They are defined as follows:

- Graphical constraint of legibility: The minimal distance must be respected between the lines so that all the information can be distinguished and the final chart is legible and clear;
- Application constraint of safety: The depth indicated by an isobathymetric area should not be deeper than the actual depth, otherwise a sailor could consider an area is deeper than actual submarine relief and navigate in a too shallow area; and
- Geomorphologic structural constraint: The relief must be preserved and the most characteristic elements must be emphasized.
Two kinds of approach are considered for line generalisation. The first approach was local, that means smoothing is done by considering few points of a line at a time. For example, in the methods presented in [14], smoothing is performed by considering sequences of several consecutive points on a line and computing an average position. To the contrary, the global approach, the second one, processes the whole curve at once. A main advantage of global approach over local one is that the entire curve is taken into account and the main characteristics are better preserved. These methods can be based on vector generalisation techniques or on the resolution of a system of equations. The former simplifies lines by selecting characteristic points. A review and evaluation of different line simplification methods is presented in [18]. The latter is based on minimisation techniques and can be used to combine several operators or several curves and therefore can take the environment into account and solve more complex problems. [11] presents a global geometrical method where generalisation is performed by using optimization techniques such as the least square method. [4] considers the curve as a snake and smooths it by reducing its energy defined from its derivatives. In [8] and [16], curve smoothing is done by viewing the polygonal line or control polygon as a cable network undergoing mechanical forces.

Compared to line generalisation for terrestrial and urban maps cases, the main issue in marine charts is the safety constraint. Indeed, methods defined for the first case cannot be applied to marine chart generalisation where specific methods must be introduced. To our knowledge, only the cable network based method introduced in [16] takes into account the safety constraint. Smoothing is done by applying forces defined by the user which tend to deform the curve while some points are fixed in order to observe the safety constraint. Although this method provides good results, it is difficult to control the smoothness of the final curve.

Furthermore, none of the smoothing methods mentioned hereabove take into account the removal of distance conflicts that already exist or are developing during the process. These conflicts can be loops or cusps or visual intersections when two distinct segments of a curve are too close. In traditional methods, when no position constraint is imposed, no loop can appear during the process and the smoothing operation iron out all protrusions on the curve. Therefore, conflict removal is not considered in these methods. In our case, it has to be tackled as many protrusions or bottle-neck shapes cannot be smoothed due to the safety constraint. As a consequence, specific techniques are needed to suppress the loops or the conflicting segments.

Snakes have been first introduced in cartographic generalisation for line displacement in [5] and have been further developed in [2]. Snakes have also been applied to polygon generalisation in [9]. In all cases, snakes were defined as polygonal lines. In their research, a constraint as strong as the safety constraint was not taken into account. Snakes for marine chart generalisation have been introduced in [10] where a displacement operator for the correction of distance conflicts based on B-spline snakes is detailed.

In the wake of these works, the current paper introduces a smoothing method based on a snake model which respects the safety constraint and takes into consideration visual conflicts (visual and real self-intersections). Isobathymetric lines are modelled with B-spline curves. The snake energy is defined according to both the smoothness of the curve and the safety constraint. In order to ensure deformation is done in the right direction, critical control points are fixed. Conflicts are corrected by removing conflicting segments and rebuilding the line. As a consequence, the two main contributions of this paper are first, the definition of a snake based smoothing method under position constraints and second, the detection and correction of visual conflicts on a curve. The paper is organised as follows: in the next section, notations and definitions are given about B-spline curves and snakes. In section 3, the smoothing method is detailed: energy expression is defined and conflict correction is tackled. Section 4 presents the global method and discusses results obtained on several case studies. Finally, conclusion and directions for future works are given.
2. PRELIMINARY DEFINITIONS

2.1 B-spline curve representation

A B-spline curve $f$ is a parametric function defined on an interval $I = [a, b] \subset \mathbb{R}$ in $\mathbb{R}^2$. It is defined by equation 1:

$$f(t) = \sum_{i=0}^{m} Q_i N_i^k(t)$$

The points $Q_i \in \mathbb{R}^2$ are the control points of the curve. The set $(Q_i)_{i=0}^{m}$ defines the control polygon of $f$. $N_i^k$ are the B-spline basis functions. They are piecewise polynomial functions of degree $k-1$ defined from $I$ to $\mathbb{R}$. To define them, we need a series of real values $(t_0 = a \leq t_1 \leq \ldots \leq t_i \leq \ldots \leq t_m + k = b)$ called the knot vector. Each basis function is positive and non zero on a local interval given by the knot vector: $N_i^k(t) = 0 \quad \forall t \notin [t_i, t_{i+k}]$. As a consequence, the curve can be controlled locally as there are no more than $k$ non zero basis functions on an interval $[t_i, t_{i+1}]$.

A main interest of the B-spline representation is that the shape and location of the control polygon is related to the shape and location of the curve so that the displacement of a control point $Q_i$ involves a local deformation of the curve on the interval $[t_i, t_{i+k}]$ and a curve segment defined on an interval $[t_i, t_{i+1}]$ is always included in the convex hull of the control points $Q_{i-k+1}, \ldots, Q_i$ (Figure 2).

![Figure 2: Control polygon (dashed line) and convex hull (in grey) of a B-spline curve.](image)

In order to compute the curve deformations and to detect visual conflicts, we need a polygonal approximation of the curve. The accuracy of the approximation is defined by a tolerance $\epsilon_{\text{num}}$. This tolerance must be small enough so that the polygon is considered as merged with the curve. This approximation is performed by computing points on the curves or by increasing the number of control points so that the control polygon converges towards the curve [7]. The approximation is expressed by a polygonal line $(P_i)_{i=0}^{m}$ and a set of parameters $(\zeta_i)_{i=0}^{m}$ so that each point $P_i$ is an approximation of $f(\zeta_i)$.

In the following, B-spline curves are defined as cubic curves ($k = 4$) with first and last knots of multiplicity $k$ (that is $a = t_0 = \ldots = t_{k-1}$ and $t_{m+1} = \ldots = t_{m+k} = b$). This convention is chosen so that the curve $f$ passes through the first and last control points. For more information about B-spline curve theory, the reader is directed to [7].

2.2 Expression of the derivatives

Two main features are used during the process to estimate the smoothness of the curve. They are the tension and the flexion. The former is given by the first derivative with respect to the curvilinear abscissa. If the derivative is globally low along the curve, it means that the curve is stretched. The tension is also related to the length of the curve as integrating the first derivative along the curve gives the length of the curve. The flexion is given by the curvature. The curvature at a point corresponds to the inverse of the radius of the osculating circle. If the curvature is low, that means that the curve orientation given by its first derivative does not vary much, so that the shape of the curve is similar to a straight line in the vicinity of the point. On the opposite, a large curvature characterises a big change of orientation of the curve. The curvature is given by the second derivative with respect to the curvilinear abscissa $s$.

A B-spline curve being a parametric curve, the derivatives are expressed with respect to the parameter $t$. The first derivatives with respect to $t$ and $s$ are equal. However, the second derivatives are different. The relation between the curvature and the derivatives is given by equation 2.

$$\kappa(t) = \frac{\det(f'(t) \times f''(t))}{\|f'(t)\|^3}$$

Approximations of the first derivative and the curvature are computed at different positions by applying finite difference schemes on the polygonal approximation of the curve.

2.3 The snake model

Snakes were first introduced by Kass et al. [12] in image processing for contour detection. They are smooth lines defined with their own energy from their geometrical features. A snake is at an equilibrium position when its global energy is minimal. In order to minimize its energy and reach this position, the snake can deform itself. Let $f(t)$ a parametric curve defined on the interval $[a, b]$. Its energy is expressed as

$$E_{\text{snake}} = \int_{a}^{b} E_{\text{int}}(f(t)) + E_{\text{ext}}(f(t)) dt$$

where $E_{\text{int}}$ is the internal energy and $E_{\text{ext}}$ is the external energy. The internal energy controls the shape of the snake and is defined from its derivatives:

$$E_{\text{int}} = \frac{1}{2} \left( \alpha(t) \|f'(t)\|^2 + \beta(t) \|\kappa(t)\|^2 \right)$$

where $\alpha$ and $\beta$ are shape parameters, usually fixed by the user. The $\alpha$ controls the tension given by the first derivative and $\beta$ controls the curvature. The external energy usually represents application constraints as it is not related to the geometry of the curve. In most of the cases, snakes are defined by polygonal lines. B-spline snakes have been used for image processing in [3] and for curve and surface approximation in [15]. The interests of B-spline snakes are that:

- less points are needed in comparison with a polygonal snake;
- smoothness is an intrinsic characteristic of B-spline curves;
- control points allow a better control of the snake.

The unknowns of the problem are the control points of the B-spline curves. The different energies are computed from the polygonal approximation of the curve. The resolution of the system is iterative. At each iteration, equation 3 is solved by applying a gradient method. This method has been chosen because curvature is a non quadratic function.
and it improves the behaviour of the model as both coordinates are solved in the same system compared to methods which solve the problem for each equation separately.

3. ISOBATHYMETRIC LINE SMOOTHING

3.1 Definition of the energies

The smoothing process includes the simplification of the shape of a curve by removing small oscillations or details in order to maintain the prime information about the shape and the orientation of the curve while removing irrelevant information. As the smoothness of a curve is related to its curvature, the curvature energy in equation 4 can be used to express the smoothness to the curve. The lower this energy, the smoother the curve.

The operation must be done respecting the safety constraint. Therefore, an external energy limiting the displacements must be added. The safety constraint is satisfied if, compared to its initial position, the smoothed curve is entirely located on the deeper side. In order to prevent wrong displacements, an energy in proportion with the distance from the initial curve is brought to the points which are on the shallower side.

\[
E_{\text{ext}}(f(\zeta)) = \begin{cases} 
\min_{0 \leq i \leq n} \left(\frac{\parallel f(\zeta_i) - f_{\text{init}}(\zeta_i)\parallel^2}{\epsilon_{\text{vis}}} \right) & \text{if } f(\zeta_i) \text{ is on wrong side} \\
0 & \text{otherwise}
\end{cases}
\]

(5)

The constant value \(\epsilon_{\text{vis}}\) corresponds to the minimal distance that defines visual intersections and is used here to normalise the energies. In the same way, the internal energy terms are also normalised so that all terms have the same magnitude. By this way, the same model can be applied to any curve with no need to adjust the parameters \(\alpha\) and \(\beta\) which are set constant.

The energy definition given in equations 3 and 5 is sufficient to solve most of the regular cases however it may not be enough to solve some singular cases with large oscillations. The curve is smoothed by applying local deformation until a stable position is found, that is, when a minimal energy is reached. In a place where the curvature is high, the displacements generated by both energies can be in opposition and a balance position can be found which does not satisfy the safety constraint. A first solution which was used in [10] for curve displacement was to choose non constant parameters \(\alpha\) and \(\beta\) in order to modify the weights between the energies. Such a solution was also used in [4] to control the importance of the smoothing. However, in the present case, increasing the importance of the external energy would limit the deformation and bring the curve back to its initial position.

A better way to prevent the displacement of control points in the wrong direction is to fix control points that are critical so that smoothing is done by displacing the other points. This technique has been introduced in [16]. Critical points are defined by applying the Douglas-Peucker algorithm [6] on the control polygon. Points are selected with a tolerance equal to \(\epsilon_{\text{vis}}\). They are the control points which define the most characteristic features of the curve and are defined by looking at the orientation of the control polygon. As the smoothing process tends to flatten the control polygon, the control points that form a protrusion towards the deeper side would be shifted to shallower areas. These points have to be fixed to limit displacements in the wrong direction (Figure 3). As these control points define the main features of the curves, by fixing them, we also preserve the main features of the curve.

![Figure 3: Left: Control polygon and fixed control points. Right: Line before and after smoothing. Units are in cm.](image)

3.2 Conflict processing

Besides the safety constraint, the generalisation process must take into account the legibility constraint. According to this, the distance between two curve segments cannot be less than \(\epsilon_{\text{vis}}\). A curve should also not admit any kind of artefact such as loop or cusp. These conflicts can either exist before the smoothing process or appear during the operation. In the first case, they can be visual intersections as the representation is not adapted to the scale or real intersections due to some numerical errors. In the second case, they appear when two segments are shifted towards the same location.

When two segments are in conflict, two cases must be distinguished. If the intersection is visual and the area between the two segments is shallower, the conflict will be solved during the smoothing process as both segments will be shifted apart towards deeper areas. In the other cases, the correction of the conflict requires a modification of the curve topology that cannot be handled by the snake method. The conflicted segments need to be removed and the curve reconstructed. As a consequence, a further step is required in the smoothing process which is the detection and the correction of the visual conflict.

Visual conflicts are detected from the polygonal approximation. Two segments \(P_n P_{n+1}\) and \(P_{n-1} P_{n}\) are in conflict if the distance between them is less than \(\epsilon_{\text{vis}}\) and if the curve turns through a total angle greater than \(\pi\) when going from one segment to the other. From this consideration, a non self-intersection criterion is deducted: if the total angle is smaller than \(\pi\), no self-intersection can occur [1]. The detection process gives us a series of conflict segments. By regrouping the conflicting segments, a line segment \(P_{n_0} P_{n_1}\)
is defined. As each point $P_i$ is associated to a parameter $\zeta_i$, the curve is in conflict on the parametric interval $[\zeta_{i_0}, \zeta_{i_1}]$.

As deformation is performed by acting on the control points, the curve has to be modified on the interval $[t_{i_0}, t_{i_1}]$ with $t_{i_0} \leq \zeta_{i_0} \leq t_{i_0} + 1$ and $t_{i_1} - 1 \leq \zeta_{i_1} \leq t_{i_1}$.

The conflict is corrected by removing the control points that define the conflicting segment and by constructing a new control polygon (Figure 5) so that the line passes through a point $P$ which is the middle of the segment $P_{i_0}P_{i_1}$ and the derivative at point $P$ is given by the direction $P_{i_0}P_{i_1}$ (Figure 4). As $P$ is the middle of the segment, the parameter $\zeta$ so that $f(\zeta) = P$ is chosen as the middle of $[\zeta_{i_0}, \zeta_{i_1}]$.

The new coordinates of the control points are obtained by solving the following system based on equation 1:

\[
\begin{align*}
  f(\zeta) &= P \\
  f'(\zeta) &= P_{i_1} + 1 - P_{i_0}
\end{align*}
\]

(6)

4. APPLICATION

4.1 Parameters and constants setting

In the previous sections, several parameters and constants are used to control the approximation and the deformation and to define the conflicts. In order to apply the method for the generalisation of a complete chart, parameters and constants have to be fixed to some values which remain unchanged during the whole process. As said before, the snake parameters $\alpha$ and $\beta$ are fixed to constant. As the smoothness is related to the curve curvature, more weight should be put on the curvature energy. However, during the process, by removing the oscillations, the curve length should be reduced. If $\alpha$ is not big enough, the curve is too loose and large deformations can occur. Different values were tested and the most satisfactory results were obtained when tension and curvature have the same importance. As a consequence, $\alpha$ and $\beta$ are fixed to 1.

Others constants are used in this algorithm which depend of the accuracy of the input and output data. The first one is $\epsilon_{num}$ which defines the numerical precision of the computation. It is particularly used to compute the polygonal approximations of the B-spline curves. This value depends of the precision of the input data, the rounding errors done during the computations and also the computation time as high precision requires more accurate approximation and more computation. The last constant $\epsilon_{vis}$ corresponds to the minimal distance to be observed between two lines. It is this value that defines visual conflicts.

In the data set on which the algorithm was tested, the points were given with an accuracy of $10^{-4}$ (all units are in centimetres). According to the cartographers’ experience, $\epsilon_{vis}$ is equal to 0.02 cm. The $\epsilon_{num}$ is set to $10^{-3}$. Compared to $10^{-4}$, this value reduces the computation time, takes into account rounding errors and is still negligible in front of $\epsilon_{vis}$.

The last point to consider is the definition of a convergence criterion. Three points must be satisfied: the safety constraint must be respected, no visual conflict should remain and the curve must be sufficiently smoothed. The two first points can be checked easily. The validation of the last point is more subjective.

The criterion should be related to the curvature as it characterises the curve smoothness. Two values are usually considered which are the maximum of curvature and the $L^2$ norm of the curvature. The first value is too local and does not reflect the global curvature of the curve. The second gives an average value of the flexion along the curve and corresponds to the curvature energy of the snake. Therefore, the line is considered smooth when its curvature energy has been significantly reduced. As the amount of
4.2 Results

The energy varies with the shape and length of the curve, setting a fixed threshold value for the curvature energy is not relevant. The problem is tackled by making the logical assumption that the energy ratio between the smoothed line and the original line should be related to the scale factor between the initial and final charts. We consider the line is smooth enough when the ratio between the initial and final energies is equal to the ratio between the scales of the initial and the final charts. This criterion is based on practical studies and should be refined in further work.

4.2.1 Results

The method has been applied on several data sets provided by the SHOM, the Hydrological and Oceanographic Service of the French Navy in charge of editing and publishing marine charts in France. Data were polygonal lines representing isobaths extracted from the SHOM bathymetric database. B-spline curves are obtained by applying a compression algorithm [16]. Different examples are presented for lines smoothed with different energy ratios. The results have been validated by cartographers from the SHOM.

In figure 7, the same line is represented at different scales. In each case, the energy ratio that defines the convergence criterion is set equal to the scale factor between the original line and the smoothed line. The three scale ratios are one half, one fifth and one tenth. The convergence criterion is consistent as the line is smoothed at different levels according to cartographer’s expectations. The main features are maintained while details such as concavities are removed in regards with the scale.

Figure 8 presents another case of smoothing. The line is not regular with many oscillations in the upper part and few but large oscillations in the lower part. The line has been smoothed by reducing the energy as in the previous example but they are shown at the same scale on the figure for comparison. When the ratio decreases, smoothing is not performed regularly along the line. When the ratio is equal to one half, the curve is smoothed by removing the small oscillations so that the curve stays close to its original position. When the ratio is equal to one fifth or one tenth, deformations are much bigger on the lower part of the line. Oscillations on the upper part have been removed and the line cannot be smoothed anymore. As a consequence, the energy is reduced by shifting the points in the lower part.

Figure 6: Line after correction. Units are in cm.

Figure 9: Smoothed lines obtained in figure 8 at the corresponding scale.

5. CONCLUSION

In this paper, a smoothing operator for generalisation of isobathymetric lines, modelled as B-spline curves, is introduced. Specific operators have to be defined for marine chart generalisation as the specific constraint of safety has to be considered. Smoothing is performed by using a snake model. The line is considered as a deformable model with its own energy, defined by its derivatives, and an external energy related to the safety constraint. The line is smoothed by reducing the total energy. In order to prevent the curve to converge towards a non feasible solution and to preserve the main features of the line, characteristic control points are fixed. The consistency of the solution is also checked during the process by detecting the occurrence of any spatial conflicts such as self-intersections. These conflicts are corrected by removing the corresponding curve segments and reshaping the curve on the interval.

The different parameters that appear in the model such as the energy parameters and the different accuracies are detailed and their value is fixed so that the smoothing process can be performed without user intervention. The only parameter that is left to user appreciation is the convergence criterion. In this paper, it is related to a percentage of the global curvature energy of the curve. If generalisation is done by passing to a smaller scale, the energy ratio is given by the scale factor between the two charts. The method has been applied to several isobaths issued from real data sets. Three examples are presented. The lines are smoothed and reduced with different scale ratios. The algorithm preserves the main features of the lines and respects the generalisation constraints. The results have been validated by cartographic practitioners.

The results shown in this paper are highly satisfactory. The convergence criterion defined here is rather subjective and has been chosen based on the results obtained on several case studies. The criterion can be improved and defined locally according to the shape of the line. In this sense, the computation time could be reduced as the algorithm would only be applied on the segments which are not smooth enough. Computation may also be limited by using some
compression techniques to reduce the amount of data. After smoothing or scale reduction, the line can be expressed with less control points, so that during the process, superfluous control points could be removed.

Another direction for future work is the extension of the snake model to other generalisation operators. A displacement operator has already been defined in [10]. Other operators such as aggregation or caricature could be defined. The objective would be to yield an operator that can combine different operations. One main difficulty is the consideration of topological transformations and other objects in the neighbourhood that may influence the result. For that purpose, other position constraints shall be imposed to check which curve should be modified and which operator can be applied.

Finally, a strategy should also be defined to perform global generalisation in a set of isobaths. The strategy must be based on the definition of spatial relationships between the isobaths and formal generalisation rules. For that purpose, a generalisation quality estimator should be introduced in order to compare different generalisation operations and to validate the results. This estimator can be based on the comparison of the surfaces defined from the isobaths [19] before and after generalisation.

6. ACKNOWLEDGMENTS

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7. REFERENCES

Self-intersection of composite curves and surfaces.
Figure 8: Line smoothing with three different energy factors (left: 1/2, middle: 1/5, right: 1/10). The dashed line is the original line. Units are in cm.