

# Precise Hong Kong Geoid HKGEOID-2000

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## Abstract:

This paper briefly discusses the data and methodology used in the construction of a Hong Kong local geoid of centimeter accuracy HKGEOID-2000. The results have been published in the website of Department of Land obtain the Surveying and Geo-Informatics, the Hong Kong Polytechnic University for public use. Users can directly geoidal height of a point by inputting its geodetic coordinates (in ITRF96 reference frame) or the HK1980 grid coordinates. Examples are given to illustrate the procedures.

## 1 Introduction

The heights of points on land are traditionally determined by leveling technique. The leveling is a time-consuming and labor-intensive operation process, though it provides precise information on the heights of points above the mean sea level. With the development of GPS survey technique the height of points can be determined with much ease. However, GPS derived heights are referred to WGS-84 ellipsoid, while the leveled heights are referred to geoid. Their difference is called geoidal height or geoid undulation. In practices, most of survey projects require leveled heights, for they have physical meaning. Therefore precise geoid must be available to transform GPS derived heights to leveled heights.

With the support of Research Grant Council (RGC) of Hong Kong Government the authors conducted a project “precise determination of Hong Kong geoid using heterogeneous data”. Its main objective is to construct Hong Kong local geoid with centimeter accuracy using the existing data and information. The project has completed with a product HKGEOID-2000. HKGEOID-2000 has one-kilometer resolution and covers the whole territory of Hong Kong, from 800km to 850km in northing and 800km to 870km in easting in the HK19980 grid coordinate system. The tests showed that it can provide geoidal height at any point within the territory with accuracy better than 1.5 cm. The product is now ready for local professional community to use.

This paper firstly outlines all aspects involved in the practical computation and evaluation of the HKGEOID-2000. Examples are then given to demonstrate the way to use the product.

## 2 Data and Information Used for the Construction of HKGEOID-2000

### 2.1 Digital terrain model:

To compute terrain effects DTM of Hong Kong and its neighboring region must be available. 16 sheets of 1:20 000 digital topographical maps covering the whole territory of Hong Kong were used to generate DTMs with resolution 100m using software 3D Analyst of ArcView GIS version 3.1. 100m resolution DTM of Shenzhen was also obtained.

### 2.2 GPS/leveling data:

55 high quality GPS/leveling stations in Hong Kong are available. Their ellipsoidal heights ( $h$ ) are referenced to WGS84 ellipsoid (ITRF 96 at 1998:121), and their leveled heights ( $H$ ) above the Hong Kong Principal Datum were determined with precise geometric leveling or trigonometric leveling. The geoidal height of a point,  $N^{obs}$  (treated as “observation”) can be computed from:

$$N^{Obs} = h - H \quad (1)$$

The estimated accuracy of so derived geoidal height is at level of 1~2 cm for 17 stations, 3~5 cm for 26 stations, and no better than 10 cm for 12 stations. Figure 1 shows the distribution of these GPS/leveling stations and their accuracy.

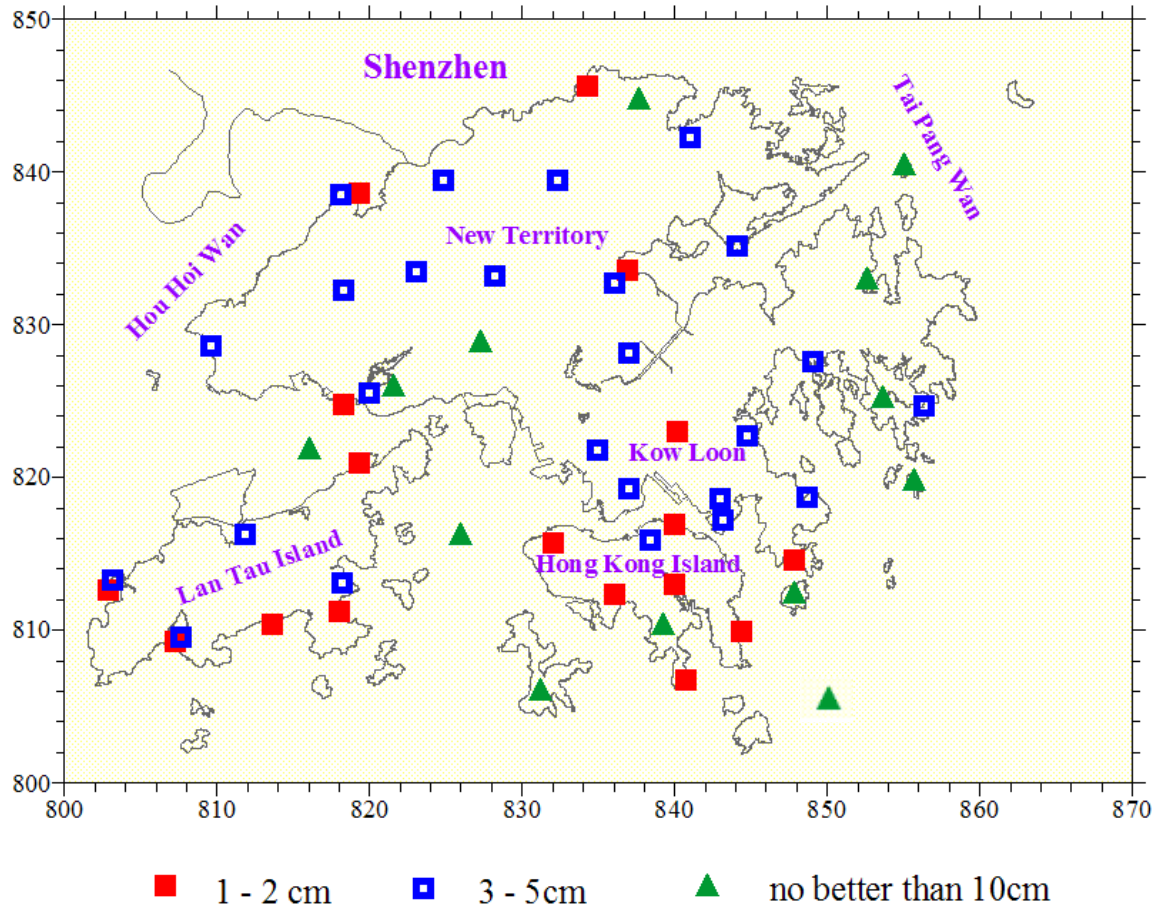


Figure 1 Distribution of GPS/leveling stations and their accuracy

### 2.3 Gravity data:

Hong Kong has 640 discrete gravity observations with station spacing 2 km on land and 2-4 km on sea. They were collected using Lacoste and Romberg model 'G' land gravity meter and model 'H/U' seabed gravity meter by Electronic and Geophysical Services Ltd. (EGS, 1988; 1991). The local gravity base is connected to International Gravity Standardization Net 1971 (IGSN 71) with accuracy of 0.03 mGal (Evans, 1990). The gravity measurements in neighboring region Shenzhen are available: 3609 points on land and 1262 points on sea with 1 km resolution. They were measured with Lacoste & Romberg model 'G' and 'D' land gravimeter and model 'S' sea gravimeter in 2001 (Ning, et al., 2003). We selected 2158 gravity measurements near Hong Kong territory together with 640 gravity measurements in Hong Kong for the determination of HKGEOID-2000. Figure 2 shows the distribution of these gravity measurements.

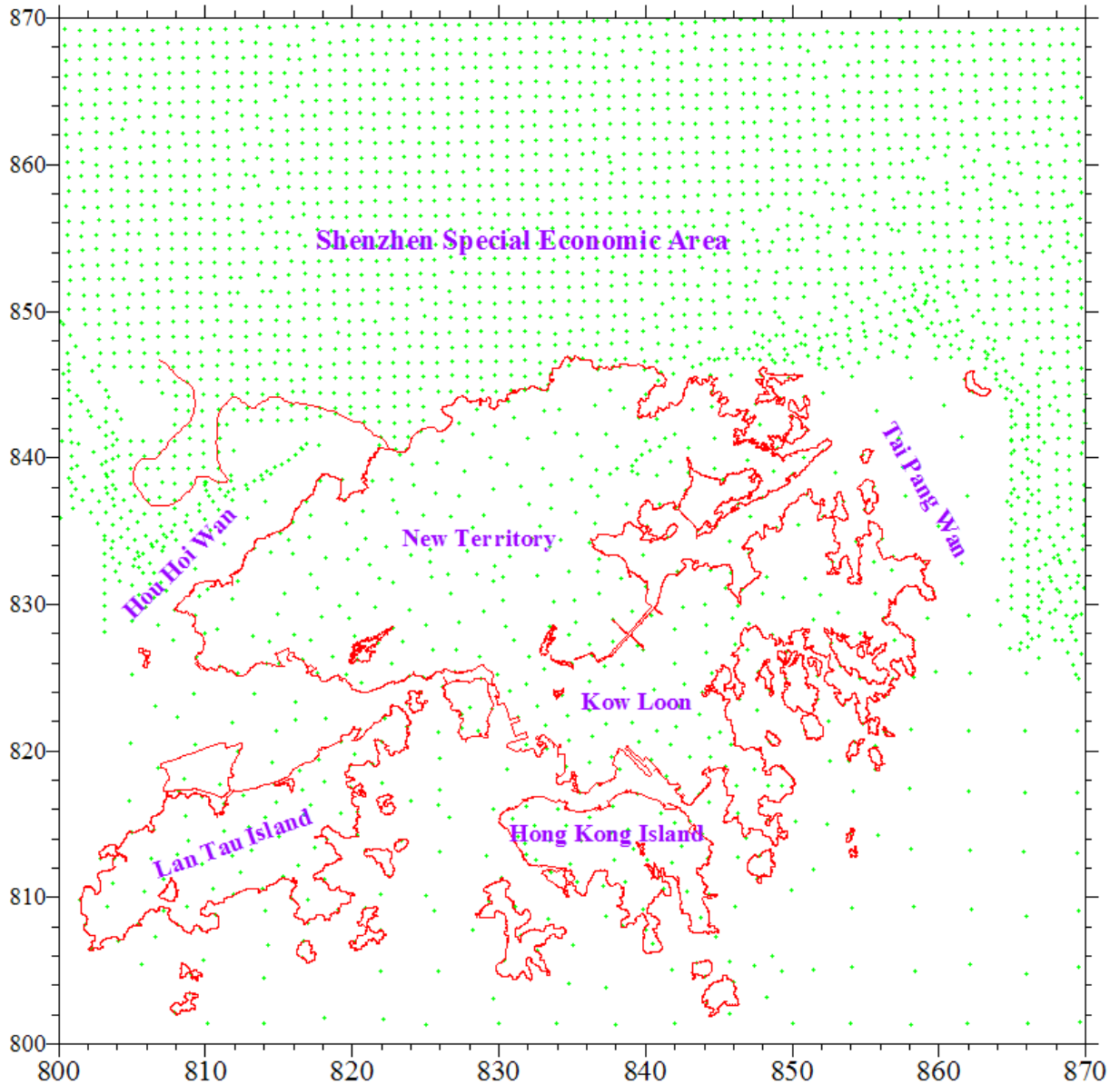


Figure 2 Distribution of gravity observations over Hong Kong and its neighboring region

#### 2.4 Selection of geo-potential model

There are several geo-potential models in the world. After a detail evaluation of these models using the data in the region, we select WDM94 for the project which is best fitted into the data. For the detail evaluation of different models the readers are referred to (Luo and Chen, 2002).

### 3 Outline of the Computation Procedure for HKGEOID-2000

A hybrid approach was used in this project. For the detail the readers are referred to (Chen and Yang, 2001; Luo and Chen, 2002).

#### 3.1 Determination of a gravimetric geoid

The remove-restore technique was used to construct the Hong Kong gravimetric geoid (Yang and Chen 2001). With the technique the long-medium wavelength component  $\Delta g^{GM}$  derived from a geopotential model and the short wavelength component  $c$  due to the topographic effect are removed mathematically from the observed gravity anomalies  $\Delta g$ ,

$$\delta g(\varphi, \lambda) = \Delta g(\varphi, \lambda) - \Delta g^{GM}(\varphi, \lambda) + c(\varphi, \lambda) \quad (2)$$

And then the Stokes' integration of the remaining parts of the gravity anomalies (called residual gravity anomaly) provides the residual geoid height  $N(\varphi_p, \lambda_p)$ . The final geoid heights of points are obtained by restoring the previous removed components, i.e., the gravimetric geoid at point  $P(\varphi_p, \lambda_p)$ , denoted as  $N^G(\varphi_p, \lambda_p)$ , and topographic indirect effect on the geoid, denoted  $N^T(\varphi_p, \lambda_p)$ , as

$$N^G(\varphi_p, \lambda_p) = N(\varphi_p, \lambda_p) + N^{GM}(\varphi_p, \lambda_p) + N^T(\varphi_p, \lambda_p) \quad (3)$$

### 3.2 Model systematic biases

The gravimetric geoid may contain systematic biases with respect to GPS/leveling derived geoid. The biases may come from the difference between the gravimetric geoid and the leveling datum, and the long-wavelength systematic errors. To remove these systematic biases a least squares fitting was carried out with the following transformation model:

$$N^{Obs}(x, y) = N^G(x, y) + a_0 + a_1x + a_2y + a_3xy \quad (4)$$

where  $x$  and  $y$  are grid coordinates of a point.

### 3.3 Refine the constructed gravimetric geoid

To improve the accuracy of the geoid, the GPS/Leveling data should be fully utilized. Therefore local Shepard surface interpolation method is used to interpolate the residuals at one-kilometer grid, i.e.,

$$\delta N(\varphi_p, \lambda_p) = \begin{cases} \sum_{i=1}^n \delta N(\varphi_i, \lambda_i) \cdot K(r_i)^m / \sum_{i=1}^n K(r_i)^m & , r_i \neq 0 \\ \delta N(\varphi_i, \lambda_i) & , r_i = 0 \end{cases} \quad (5)$$

where  $\delta N$  is the difference between the GPS/leveling derived geoid height and the transformed gravimetric geoid height,  $r_i$  is the distance between interpolating point  $(\varphi_p, \lambda_p)$  and known point  $(\varphi_i, \lambda_i)$ ,  $m$  the fitting power should be integer ( $m = 1, 2, \dots$ ), and  $K(r_i)$  the kernel function or so-called weighting function, i.e.,

$$K(r) = \begin{cases} 1/r & , 0 < r \leq D/3 \\ \frac{27}{4D} \left(\frac{r}{D} - 1\right)^2 & , D/3 < r \leq D \\ 0 & , r > D \end{cases} \quad (6)$$

with  $D$  interpolating or searching radius. The final geoid was obtained by adding the residuals at one-kilometer grid to the above transformed geoid. Such derived geoid is called hybrid or combined geoid, which named HKGOID-2000. Figure 3 shows the HKGEOID-2000.

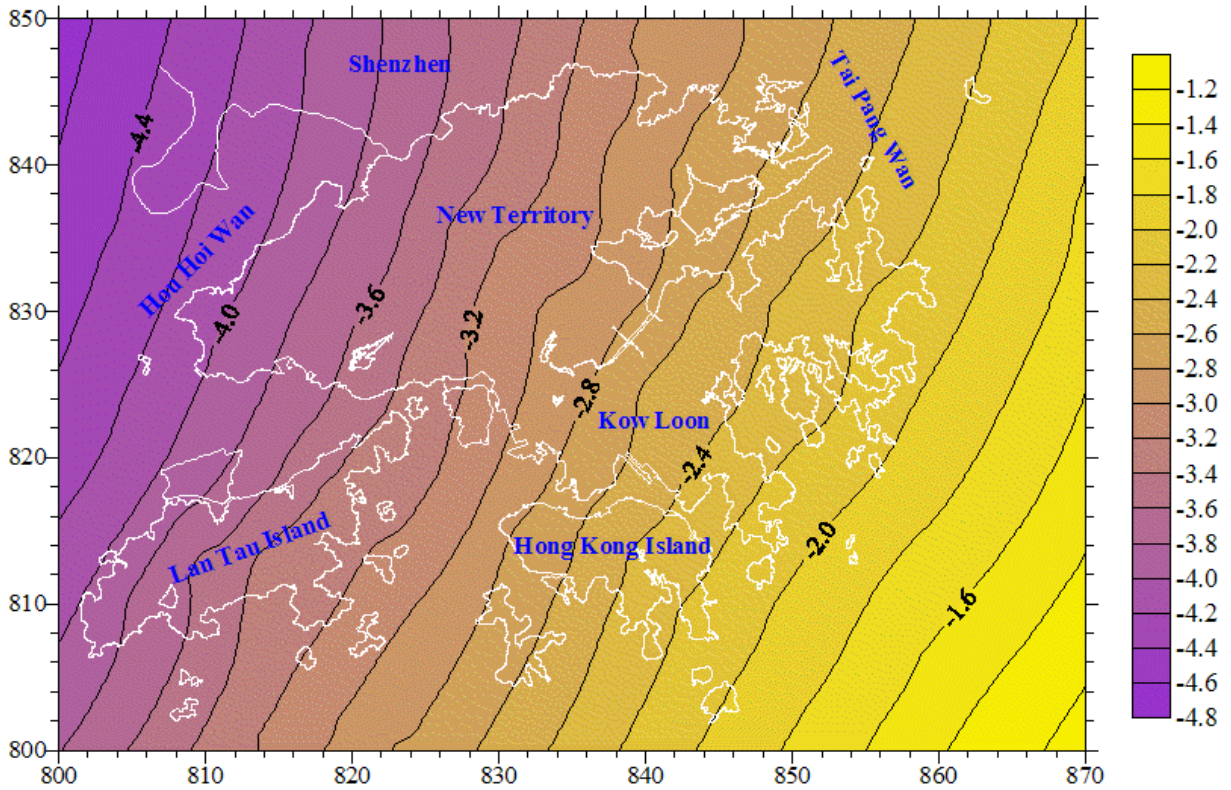


Figure 3 Contour map of HKGEOID-2000 at 1km grid  
(HK1980 Grid Coordinates System, contour interval 0.2m)

#### 4 Evaluation of HKGEOID-2000

23 precise GPS/leveling stations, which not used in the construction of HKGEOID-2000, were employed to assess the accuracy of HKGEOID-2000. The estimated accuracy of the geoid heights derived from these GPS/leveling data is better than 2 cm. Figure 4 shows the distribution of these GPS/leveling stations. The accuracy of HKGEOID-2000 is evaluated by comparing the geoid heights at these stations derived from the model  $N^{Model}$  with that derived from GPS/Leveling  $N^{Obs}$  in absolute or relative sense, i.e.

$$\Delta N = N^{Obs} - N^{Model} \quad (7)$$

and

$$\delta N = \Delta N^{Obs} - \Delta N^{Model} \quad (8)$$

The comparison shows that the root mean square (RMS) of  $\Delta N$  is 2.2 cm, and that of  $\delta N$  is better than 1 ppm for base line length longer than 25 km. Taking into consideration of the geoid heights derived from the GPS/leveling data, the practical accuracy of HKGEOID-2000 should be better than 1.6 cm (assuming that the errors of the data and geoid model have similar size), and relative accuracy better than 1 ppm in 20 km.

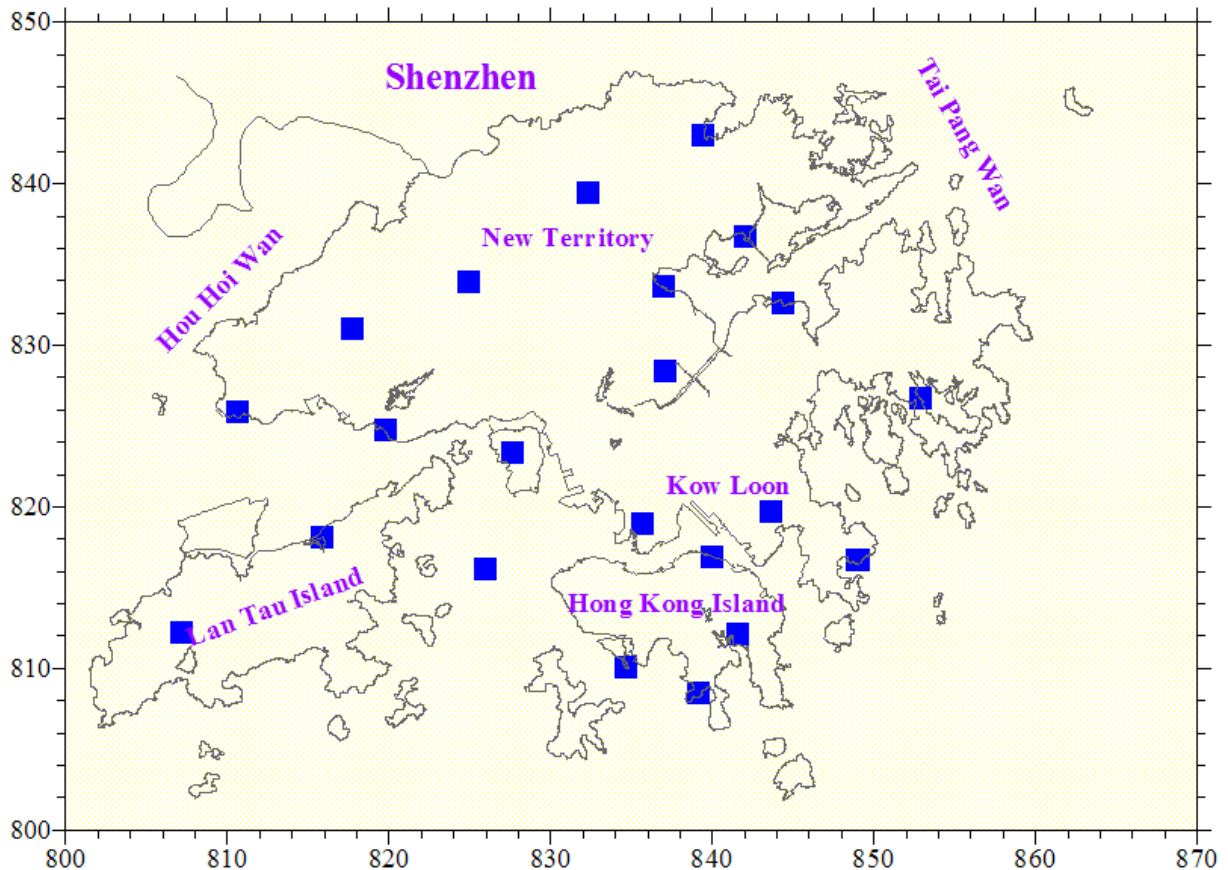


Figure 4 Distribution of 23 GPS/leveling check stations

## 5 Application

The constructed Hong Kong geoid has been published in the following website for public use. Users input the position of a point in either geodetic coordinates or the HK1980 grid coordinates and obtain geoid height as output.

[http://www.lsgi.polyu.edu.hk/project\\_HKGEOID-2000/index.htm](http://www.lsgi.polyu.edu.hk/project_HKGEOID-2000/index.htm).

*Example 1.* Given the geodetic coordinates (ITRF96 reference frame) of a point: Latitude = 22 31 36.77259, Longitude = 114 12 27.86855, Ellipsoidal height  $h = 35.413$ , compute the leveled height.

Go to the website and click “get the geoid“, then input the latitude and longitude values. It gives you geoid height  $N = -2.972\text{m}$ . The leveled height of the point  $H$  is computable from

$$H = h - N = 35.413 - (-2.972) = 38.385 \text{ m}$$

*Example 2.* Given the HK1980 grid coordinates of a point as  $E = 850000.000\text{m}$ ,  $N = 830000.000\text{m}$ . Get its geoid height.

$$\text{The result } N = -2.344\text{m}$$

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