Algebraic Models for Feature Displacement in the Generalization of Digital Map Data Using Morphological Techniques

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Abstract Feature displacement is one of the many operations (or operators) in map-data generalization. It has become a priority item on the research agenda. This paper describes some algebraic models (or mathematical models) for this operation. These models are based on the operators developed in mathematical morphology, which is a science dealing with form, shape, and structure of objects.

Feature displacements can be classified into two groups, i.e., feature translation and feature modification. For the first type, a set of structuring elements (key elements in morphological operators) are developed. Using this set of structuring elements, the two basic morphological operators, i.e., dilation and erosion, can be used for translating features in any of the eight directions, freely. Translation of features in other directions can be achieved by a combination of these eight directions. For the second type of feature displacement, a number of models have been developed to suit different cases, i.e., an area feature and a linear feature, two linear features, etc. These models have also been tested using various examples. The results show that these techniques are very promising.

Introduction

GENERALIZATION is a fundamental function for geographical modelling, for efficient derivation and updating of small-scale maps and spatial databases from large scale sources, and for real-time visualization and analysis of spatial data in a GIS. Therefore, generalization is an important issue in many disciplines such as GIS, cartography, and spatial (geographical) analysis.

Generalization functionality has become increasingly important in the digital era with the widespread use of GIS. As pointed out by Abler (1987), generalization and overlay are the two "most exciting capacities" of a GIS. It

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has also been argued by Robinson (1993) that "generalization is one of the most important factors in the efficient and effective visualization of spatially referenced data." As has been emphasized by Müller et al. (1993), "Generalization facilities must be provided by GIS to support the use of geographical information at multiple scales for multiple purposes and tasks."

Indeed, generalization is so important and difficult a topic that it has nowadays become a major international research theme in cartography and geographic information sciences. Since the late 1980s, generalization has become part of the international research agenda in the spatial information sciences (Marble 1984; Abler 1987; Rhind 1988; Müller 1991). Over the last decade, many projects have been initiated internationally—in Canada, China, Britain, France, Germany, the Netherlands, Sweden, Switzerland, and the USA. For example, generalization of cartographic data and other spatial information has become one of the initiatives of the National Center for Geographic Information Analysis (NCGIA), under the title of "Multiple Representations." Also, the European GISDATA Task Force, which can be considered as a European analogy to NCGIA, has identified generalization as a key issue in GIS. In the cartographic community, the International Cartographic Association (ICA) has recently established a working group on this subject.

Over the last three decades, many generalization operations, such as selection, omission, combination, aggregation, coarsen, collapse, and displacement, have been identified by researchers (e.g. Rhind 1973; Robinson et al. 1978; Brassel and Weibel 1988; Keates 1989; Shea and McMaster 1989; Beard and Mackaness 1991). However, the current situation is that most of the generalization operations remain at the conceptual level. In other words, there are no algorithms or mathematical models developed for them. Indeed, "generalization procedures to transform the information content of a map from one scale to another are notably absent within the realm of functions usually available in GIS," as pointed out by Müller (1989). To improve the situation, the ICA Working Group on Automated Map Generalization, led by Professor R. Weibel of the University of Zurich, has recently prioritized the develop-

ment of those "missing" algorithms, or mathematical models, for generalization operations (Weibel 1995). This paper describes a set of mathematical models for one of these operations, i.e., feature displacement.

Feature displacement, a complex problem, has been tackled by only a few researchers (e.g. Monmonier 1987). Feature displacement can be carried out in either vector mode or raster mode. Monmonier (1987) has considered both modes. It is the authors' understanding that it should be more convenient to carry out generalization operations in raster mode, since generalization is caused by a reduction in space when scale is reduced, and raster is a space-primary data structure. As a consequence of this reasoning, the models developed in this study are in raster mode. More exactly, they are based on operators developed in mathematical morphology—a science dealing with the shape, form, and structure of objects.

This introduction will be followed by a brief discussion of the displacement problem in digital generalization; then, some operators and algorithms developed in mathematical morphology will be introduced. After that, a description of the mathematical models developed for feature displacement, and based on these morphological techniques, is given.

Feature Displacement: Problems and Possible Solutions

From the literature, it can be noted that the whole generalization process can be analyzed into a number of operations (or operators). Various sets of generalization operations have been identified by researchers. Table 1 lists some of them. In these various sets, some have more operations than others, and the same terminology may mean different things in different sets. One particular operation among many is called feature displacement.

FEATURE DISPLACEMENT: PROBLEMS

Feature displacement becomes necessary when either of the following situations occurs:
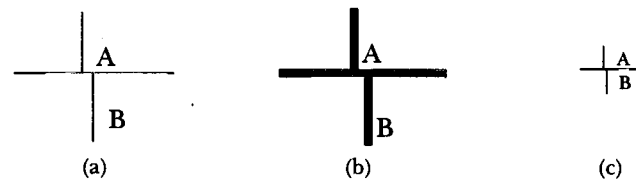1) There is a need for exaggeration of some particular

Figure 1. Simple scale reduction of a map consisting of a road junction.
(a) A road junction at large scale, with Lines A and B not aligned.
(b) Lines A and B are prepared (thickened) for scale reduction.
(c) The characteristics of the road junction are no longer clear after scale reduction, although there are still no spatial conflicts. (Modified from Keates, 1989).

characteristics.
2) There is a spatial conflict between two features.

Feature exaggeration is usually accompanied by feature displacement. Figure 1 and Figure 2 illustrate the need. This example shows the necessity of feature displacement in order to preserve the characteristics of the road junction after a scale reduction.

Spatial conflicts include crossing, touching, and coalescing. Crossing is mainly caused by generalization operations. For example, if one applies the Douglas-
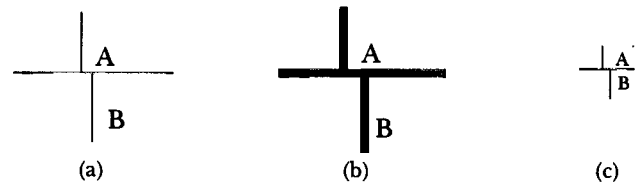

Figure 2. Scale reduction of a map showing a road junction, with exaggeration. (Modified from Keates, 1989).
(a) A road junction at large scale, with Lines A and B not aligned.
(b) Lines A and B are displaced so that the junction is exaggerated.
(c) The characteristics of the junction are retained after scale reduction.

Table 1. Generalization operations.

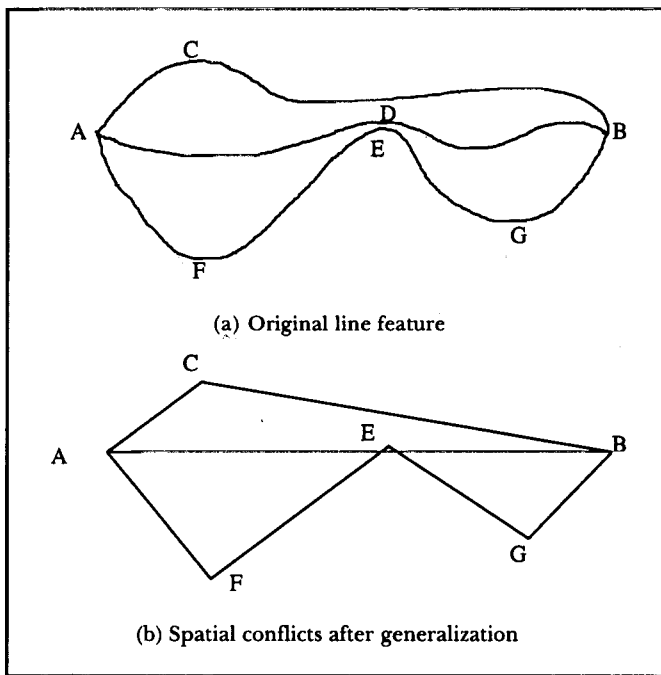| Proposers	Set of Operators
Keates (1989)	Selective omission, Simplification, Combination, Exaggeration, Displacement
Robinson et al (1978)	Simplification, Classification, Symbolization, Induction
Rhind (1973)	Line sinuosity reduction, Feature transportation, Amalgamation, Elimination, Graphic coding
Beard and Mackaness (1991)	Selection, Omission, Coarsening, Collapsing, Combination, Classification, Exaggeration, Displacement
Shea and McMaster (1989)	Simplification, Smoothing, Aggregation, Amalgamation, Merging, Collapse, Refinement, Typification, Exaggeration, Enhancement, Displacement, Classification

(a) Original line feature

(b) Spatial conflicts after generalization

Figure 3. Possible spatial conflicts created by inadequate line-generalization algorithms.

(a) Translation of both lines   (b) Translation of one line only

(c) Modification of both lines   (d) Modification of one line only

Figure 5. Two types of displacement: translation and modification.

Peucker algorithm for line generalization, one may get spatial conflicts such as those shown in Figure 3. That is because the algorithm doesn't take into consideration scale and space. That is to say, such problems are created by ourselves, and should be avoided by using more appropriate algorithms. For example, the line-generalization algorithms based on the natural principle (Li and Openshaw 1993), developed by Li and Openshaw (1992), guarantee that no such spatial conflicts will be created.

On the other hand, coalescence and touching between features are caused mainly by the nature of scale reduction. Such problems cannot be avoided, and thus need to be resolved. In fact, touching can be considered as a special case of coalescence. Figure 4 shows one of many possible cases, in which the two linear features are too close together, and need to be displaced.

FEATURE DISPLACEMENT: TWO SOLUTIONS—TRANSLATION AND MODIFICATION

For the problem shown in Figure 1 and Figure 2, a simple translation is sufficient. However, the problem shown in Figure 4 is more complex. There are two basic solutions: one is to translate one or both features in certain directions; the other is to modify one or both features. Figure 5 shows these solutions.

There are two basic types of displacement, i.e., translation and modification. Also, the operation can be applied to both features, or to one feature only. Of course, there is a possibility of a third type, i.e., a combination of translation and modification. For example, if the separation between the two solid lines in Figure 5(d) is not large enough, then a translation may be applied to one of them.

Feature translation is very important, especially in the case of exaggeration, as shown in Figure 2. To translate a feature means to move the feature to the left, to the right, downwards, upwards, to the upper left, to the upper right, to the lower left, or to the lower right. Movement in other directions can be achieved by a combination of movement in these eight directions. Feature translation is a global operation, by the same magnitude. On the other hand, modification of a feature means to move the feature partially, with the distance of movement varying from point to point. Modification is much more complex than translation.

In this paper, some mathematical models for solving the problems of both translation and modification will be described. The techniques will be in raster mode. If the original data is in vector format, then one needs to (1) rasterize the data, (2) perform the displacement operation, and (3) vectorize displaced features.

Mathematical Background: Morphological Operators and Algorithms

To build mathematical models for the displacement of features means to build models for the operation upon the two basic operators developed in mathematical

Figure 4. Two coalesced linear features to be displaced.

morphology, i.e., dilation and erosion, which can be compared to "+," "−," "x," and "÷" in ordinary algebra. In order to facilitate the discussion of those mathematical models developed by the authors, the basic concepts and relevant algorithms in mathematical morphology are introduced here.

BASIC MORPHOLOGICAL OPERATORS

Mathematical morphology is a science of form and structure, based on set theory. It was developed by the French geostatistical scientists G. Matheron and J. Serra in the 1960s (Matheron 1975; Serra 1982). Since then, it has found increasing application in digital-image processing. Efforts have also been made by researchers to apply morphological tools to mapping-related sciences, such as digital terrain modelling (Li and Chen 1991).

The two basic operators are defined as follows (Serra 1982; Haralick et al. 1987):

Dilation: $A \oplus B = \{a + b: a \in A, b \in B\} = \cup_{b \in B} A_b$ (1).

Erosion: $A \ominus B = \{a: a + b \in A, b \in B\} = \cap_{b \in B} A_b$ (2).

where A is the image to be processed, and B is the structuring element, which can be considered as an analogy to the kernel in convolution operations. In Equation 1, the process is called "dilation of A by B," and in Equation 2, "erosion of A by B." Examples of these two operators are given in Figure 6, where the features are represented by black pixels. The origin of a structuring element is at its geometric center if there is no specific indication otherwise. This convention will be followed throughout this paper.

The structuring element is the critical element in a morphological operation. It could take any shape (e.g., square or cross) and size (e.g., 2 × 2 or 3 × 3). Figure 7 shows some of the possible shapes, i.e., circular, diagonal, linear, squared, triangular, etc.

If a symmetric structuring element, such as the one shown in Figure 7 (d) or Figure 8 (b), is used for dilation, then the shape of the original image will be expanded uniformly in all directions. The dilation in this particular case is called expansion. Similarly, the erosion in this case is called shrink. These two special operations are illustrated in Figure 8(c) and Figure 8(d).
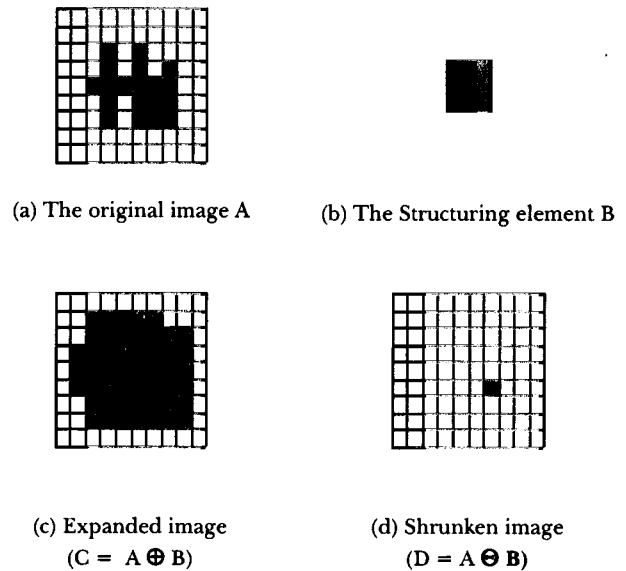
(a) Original image A

(b) The structuring element B

(c) A dilated by B
(A ⊕ B)

(d) A eroded by B
(A ⊖ B)

Figure 6. Two basic morphological operators: dilation and erosion.

(a) Circular   (b) Diagonal   (c) Linear

(d) Squared   (e) Cross
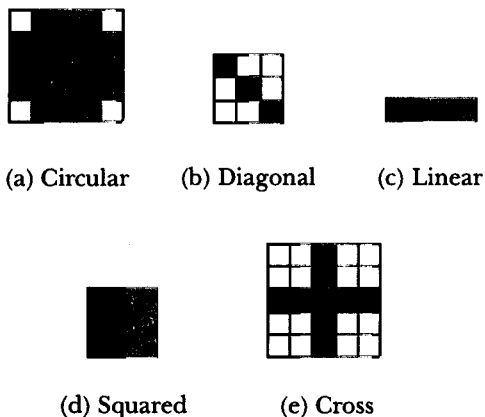
Figure 7. Some possible structuring elements.

(a) The original image A

(b) The Structuring element B

(c) Expanded image
(C = A ⊕ B)

(d) Shrunken image
(D = A ⊖ B)

Figure 8. Expansion and shrink: special cases of dilation and erosion.

THE HIT-MISS OPERATOR

Based on these two basic operators, i.e., dilation and erosion, a number of new operators have also been developed. Examples are: closing, opening, thinning,

erosion, conditional thinning, conditional thickening, sequential dilation, conditional sequential dilation, and so on.

The hit-miss operator is the basic tool for shape detection and construction in morphological algorithms. It will also be used in the models described in this paper. It is defined as follows:

$$A \otimes B = (A \ominus B_1) \cap (A^c \ominus B_2) \qquad (3),$$

where A is an original feature; A^c is a complement (or background) of A; and B is a structuring element pair, i.e., $B = (B_1, B_2)$: one to probe the inside and one to probe the outside of the feature. A point is obtained in the hit-miss output if and only if B_1, translated to the point, fits inside A, and B_2, translated to the point, fits outside A. It is assumed that B_1 and B_2 are disjoint, i.e., that $B_1 \cap B_2 = \varnothing$. The process of the hit-miss operator is illustrated in Figure 9.

(a) Original feature A

(b) Structuring element B

(c) A ⊗ B

(d) A - (A ⊗ B)

Figure 10. The process of a thinning operation.

by Equation 4. The process is illustrated in Figure 10.

$$A \ O B = A - (A \otimes B) = A \cap (A \otimes B)^c \qquad (4).$$

In practice, a more useful expression for thinning a feature, say A, (more than once) symmetrically, is based on a sequence of structuring elements, which are described by Equation 5 and illustrated in Figure 11.

$$\{B_i\} = \{B_1, B_2, B_3, \cdots B_{n-1}, B_n\} \qquad (5),$$

where B_i is a rotation of B_{i-1}.

(a) Original image A

origin B   B origin

(b) Structuring elements
B = (B_1, B_2)

(c) A is eroded by B_1
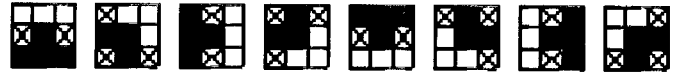
Figure 11. A sequence of structuring elements for systematic thinning ("x" means "don't care").

(d) A^c: Complement of A

(e) A^c is eroded by B_2

(f) Final result: (c) ∩ (e)

Figure 9. The process of the hit-miss operation.

In Figure 9, the hit-miss operator is used to detect and extract a feature with a "Γ" shape. Figure 9 (a) shows the original feature, and Figure 9 (f) shows the result. The hit-miss operator is so important that it is involved in most morphological algorithms.

THE THINNING ALGORITHM

There are many morphological algorithms developed for different usages. In this paper, those algorithms relevant to the models to be described are set out in this section. They are algorithms for thinning, skeletonization, and pruning.

The thinning of a feature, A, by a structuring element, B, using the hit-miss operator can be described

Using this sequence of structuring elements, the process of systematic thinning can be described by Equation 6 (Serra 1982):

$$A \ O \ \{B_i\} = ((\cdots((A \ O B_1) \ O B_2) \cdots) \ O B_n) \qquad (6).$$

This sequence of structuring elements is called homotopic structuring elements (Golay 1969). Accordingly, the algorithm described by Equation 6 is also named the homotopic sequential thinning algorithm. This thinning algorithm can be used for skeletonization of a feature, and will be illustrated in the next subsection.

THE SKELETONIZATION ALGORITHM

The skeleton of a feature plays an important role in raster-data processing. Many algorithms for skeletonization have been developed by researchers in the area of mathematical morphology. They include Lantuéjoul's skeletonizing algorithm (Lantuéjoul 1978)

Lantuéjoul's skeletonizing algorithm (Lantuéjoul 1978) based on erosion and opening; Meyer's conditional bisector algorithm (Meyer 1979); the homotopic sequential thinning algorithm; and the sequential thinning algorithm (Serra 1982). It has been found that the homotopic sequential thinning algorithm is most suitable for the purpose of this study, and it is described here.

The homotopic sequential thinning algorithm for skeletonization of a feature, say A, is defined as follows:

$$S(A) = A \bigcirc \{B_i\}. \tag{7}$$

where B_i is the set of sequential structuring elements shown in Figure 11. It is notable that the right side of Equation 7 is identical to Equation 6. This means that the skeleton, $S(A)$ of A, is obtained by the homotopic sequential thinning algorithm defined by Equation 6. The process is illustrated in Figure 12.

PRUNING ALGORITHM

After this skeletonization process, some parasitic branches on the right-most and left-most parts of the skeleton may be produced, as shown in Figure 12 (i). They need to be removed. The process for such an operation, called "pruning," can be described by Equation 8.

$$P(X) = X \bigcirc \{P_i\} \tag{8}$$

This, too, is essentially a thinning operator. The only difference is that a set of specially designed structuring elements $\{P_i\}$, as shown in Figure 13, is used in the process to allow this thinning operation to prune parasitic branches. This set of sequential structuring elements is used to eliminate the branches in all the eight basic directions, i.e., right, left, up, down, upper-right, upper-left, lower-right and lower-left. The function of each structuring element is listed in Table 2.

The pruning effect is achieved by applying Equation 8 N times with this set of structuring elements. For example, the parasitic branches shown in Figure 5 (i) are removed after applying this algorithm twice. The result is shown in Figure 14.

(a) Original feature A   (b) $T_1 = A \bigcirc B_1$ (c) $T_2 = T_1 \bigcirc B_2$

(d) $T_3 = T_2 \bigcirc B_3$ (e) $T_4 = T_3 \bigcirc B_4, B_5, B_6, B_7, B_8$ (f) $T_5 = T_4 \bigcirc B_1$

(g) $T_6 = T_5 \bigcirc B_2$ (h) $T_7 = T_6 \bigcirc B_3, B_4, B_5, B_6, B_7, B_8$ (i) $T_8 = T_7 \bigcirc B_1$

Figure 12. The process of the homotopic sequential thinning algorithm for feature skeletonization.

(a) The Original feature   (b) The result after pruning
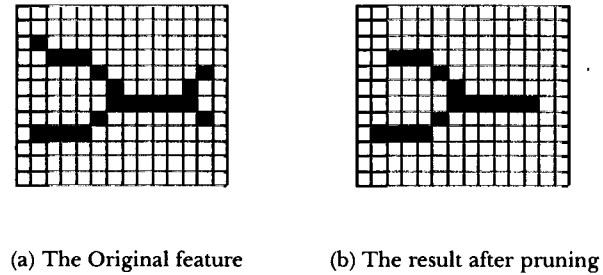
Figure 14. The effect of the pruning process.

(a)   (b)   (c)   (d)   (e)   (f)   (g)   (h)

Figure 13. A set of sequential structuring elements for pruning parasitic branches ("x" means "don't care").

Algebraic Models for Feature Translation

As pointed out previously, the structuring element is the key element in a morphological operator. Using the same morphological operator, but different structuring elements, one would obtain very different results. In this section, it will be demonstrated that, with some specially

Table 2. Functions of structuring elements for the pruning process, as shown in Figure 13.

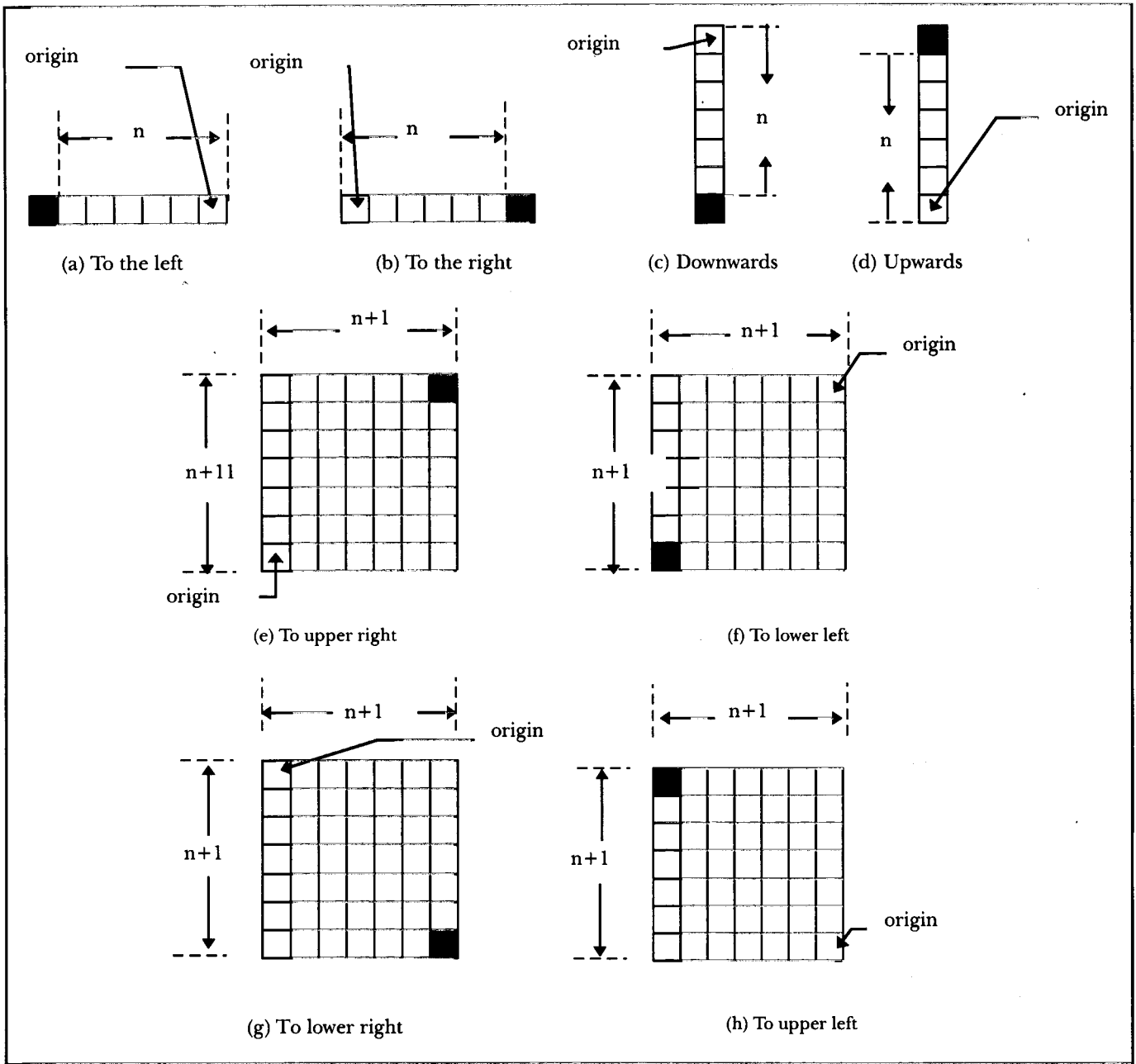| Structuring element	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Direction of effect	Right	Lower	Left	Upper	Lower-right	Lower-left	Upper-left	Upper-right

Figure 15. Structuring elements for translating objects by n pixels in 8 basic directions using the dilation operator.

designed structuring elements, simple dilation and erosion can be used to achieve feature translation.

There are two approaches for feature translation using morphological techniques. The first is through dilation and then erosion, using different structuring elements. The second is a one-step dilation (or erosion). However, the structuring elements are different in these two approaches. The displacement of a linear feature by the first approach, as illustrated by Li (1994), is in fact less efficient than the second approach, and will not be discussed here. The second approach will be illustrated.

FEATURE TRANSLATION
USING THE DILATION OPERATOR

As the title of this subsection implies, the model for translation of features, using dilation, is as follows:

$$D = A \oplus T_d \qquad (9),$$

where A is an original feature and T_d is one of the specially designed structuring elements. These elements are shown in Figure 15.

Figures 16 through 23 illustrate the translation of a linear feature in the following eight directions: right, left,
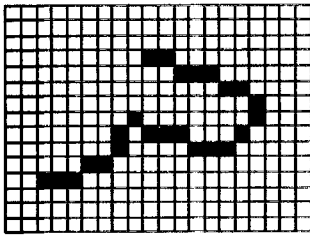
(a) The Original image A
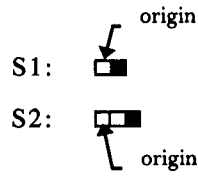
(b) The structuring elements

S1:

S2:
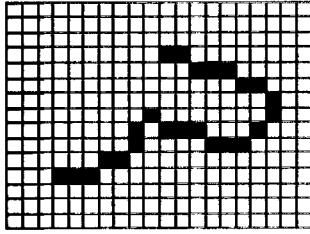
(c) Translation by one pixel
(A ⊕ S1)
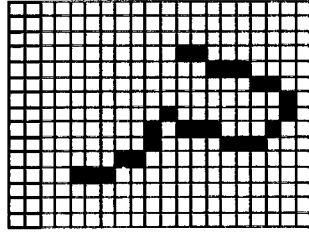
(d) Translation by two pixels
(A ⊕ S2)

Figure 16. Feature translation: to the right.
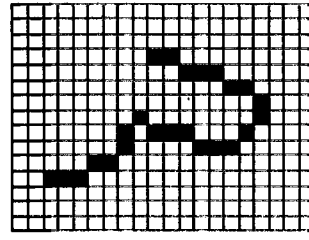
(a) The Original image A
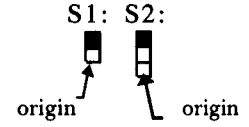
(b) The structuring elements

S1: S2:
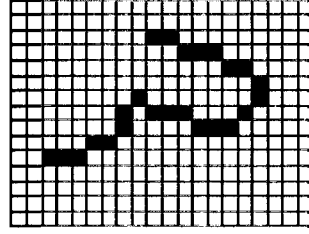
origin   origin

(c) Translation by one pixel
(A ⊕ S1)
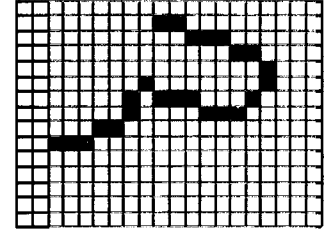
(d) Translation by two pixels
(A ⊕ S2)

Figure 18. Feature translation: upwards.
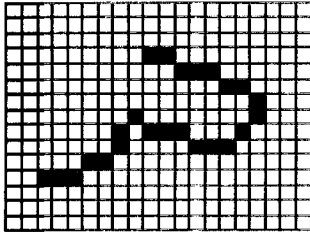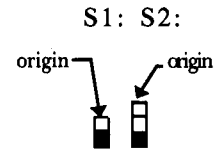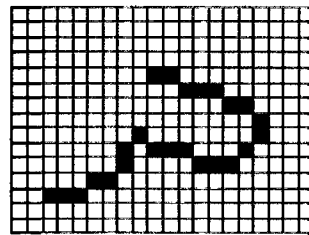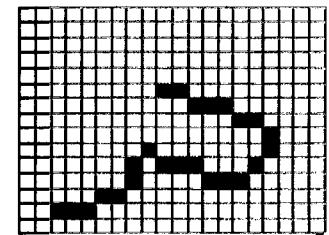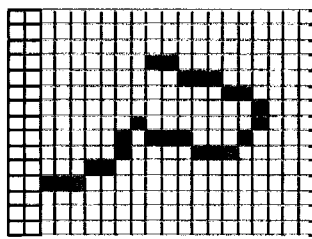
(a) The Original image A
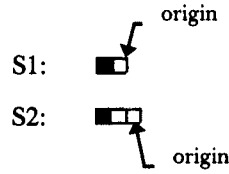
(b) The structuring elements

origin

S1:

S2:
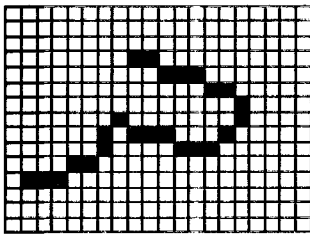
origin

(c) Translation by one pixel
(A ⊕ S1)
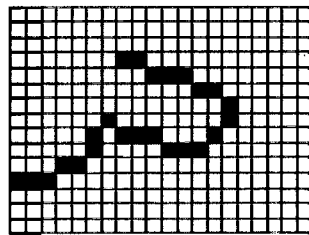
(d) Translation by two pixels
(A ⊕ S2)

Figure 17. Feature translation: to the left.
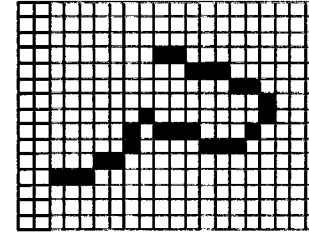
(a) The Original image A

(b) The structuring elements

S1: S2:

origin   origin

(c) Translation by one pixel
(A ⊕ S1)

(d) Translation by two pixels
(A ⊕ S2)

Figure 19. Feature translation: downwards.

up, down, upper/right, upper/left, lower/left, lower/right.

In these figures, translation of linear features by one and by two pixels is shown. However, this doesn't mean that one can translate a feature by only up to two pixels.
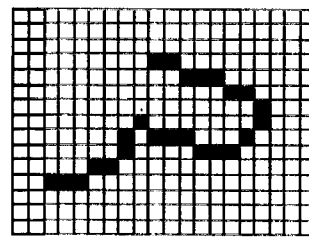
In fact, one can translate a feature with any magnitude, to any of these eight directions, as one wishes. If one wants to have a feature translated by a larger magnitude, then the size of structuring elements required will be larger,
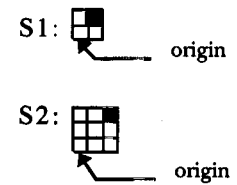
(a) The Original image A
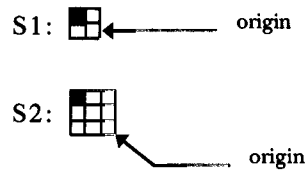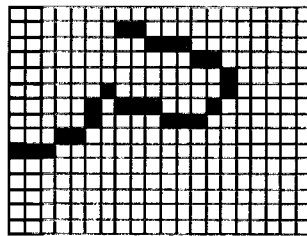
(b) The structuring elements
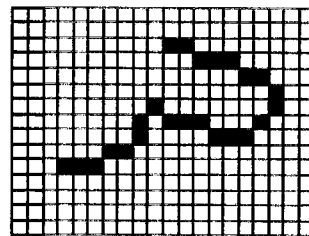
(c) Translation by one pixel
(A ⊕ S1)
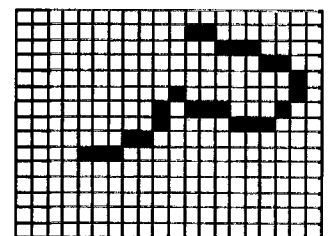
(d) Translation by two pixels
(A ⊕ S2)

Figure 20. Feature translation: to the upper left.

(a) The Original image A

(b) The structuring elements

(c) Translation by one pixel
(A ⊕ S1)

(d) Translation by two pixels
(A ⊕ S2)

Figure 21. Feature translation: to the lower left.

(a) The Original image A

(b) The structuring elements

(c) Translation by one pixel
(A ⊕ S1)

(d) Translation by two pixels
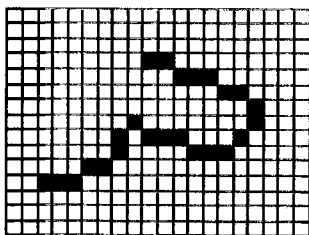(A ⊕ S2)

Figure 22. Feature translation: to the upper right.

(a) The Original image A

(b) The structuring elements

(c) Translation by one pixel
(A ⊕ S1)

(d) Translation by two pixels
(A ⊕ S2)

Figure 23. Feature translation: to the lower right.

as indicated in Figure 15.

Translation of features in other directions can be achieved by using a combination of these eight basic operations.

FEATURE TRANSLATION
USING THE EROSION OPERATOR

In fact, the same effect of feature translation can also be achieved by the erosion operator. The model is simply as

Figure 24. Structuring elements for translating objects by n pixels in 8 basic directions using the erosion operator.
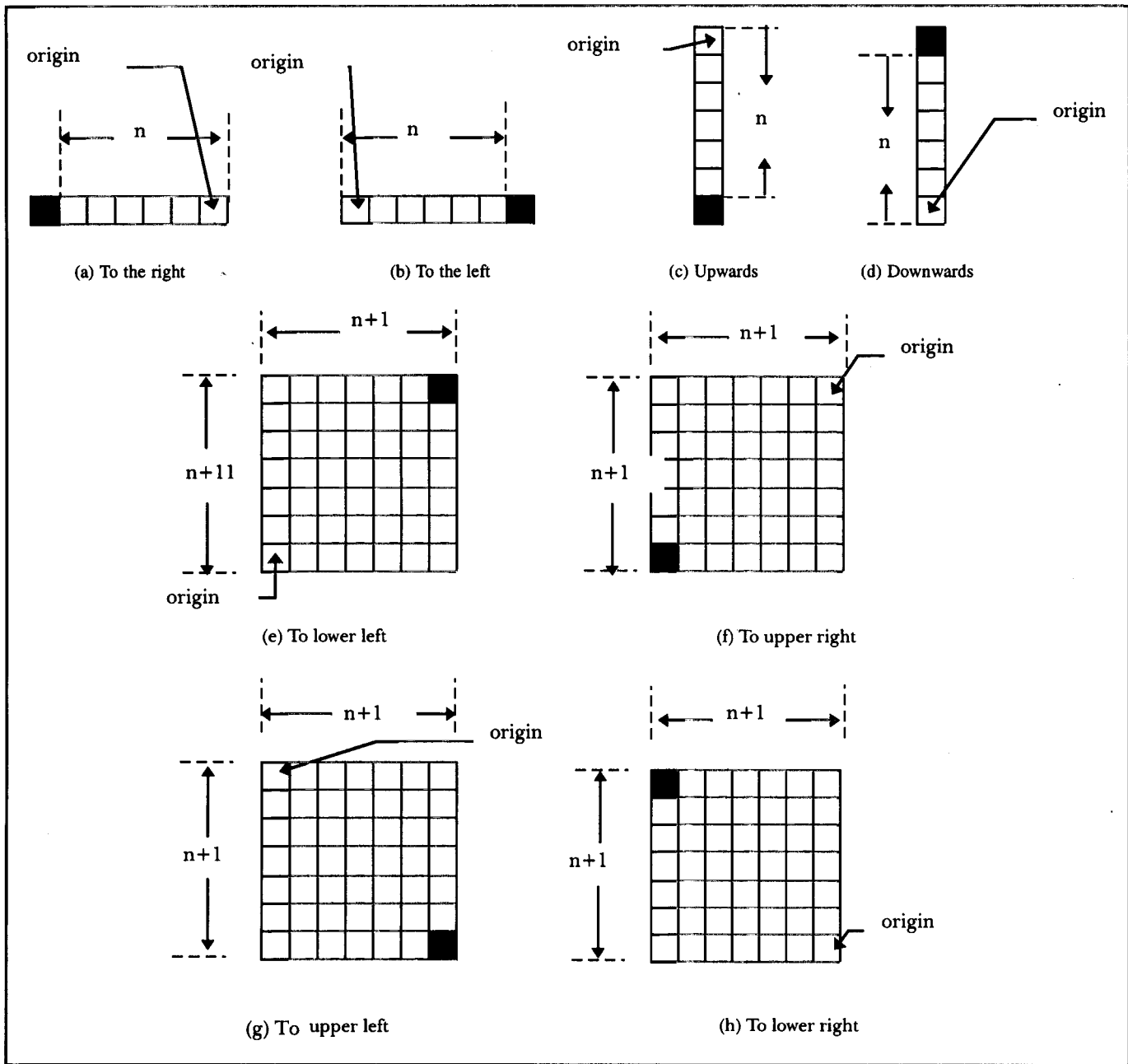
follows:

$$D = A \ominus T_e \quad (10),$$

where A is an original feature and T_e is one of the specially designed structuring elements (Figure 24).

It can be noted that this set of structuring elements is exactly the same as the set for dilation shown in Figure 15 except that the directions of movement are just the opposite. For example, in the case of dilation, the structuring element shown in Figure 15 (a) is used for translating features to the left, while, in the case of erosion, it is used

for translating features to the right.

Figure 25 shows some of the results that illustrate the translation of a feature in the eight directions by the erosion operator. The corresponding structuring elements are shown in Figure 26.

Algebraic Models for Feature Modification

With the morphological operators and algorithms introduced in the section on mathematical background, it is possible to modify features.
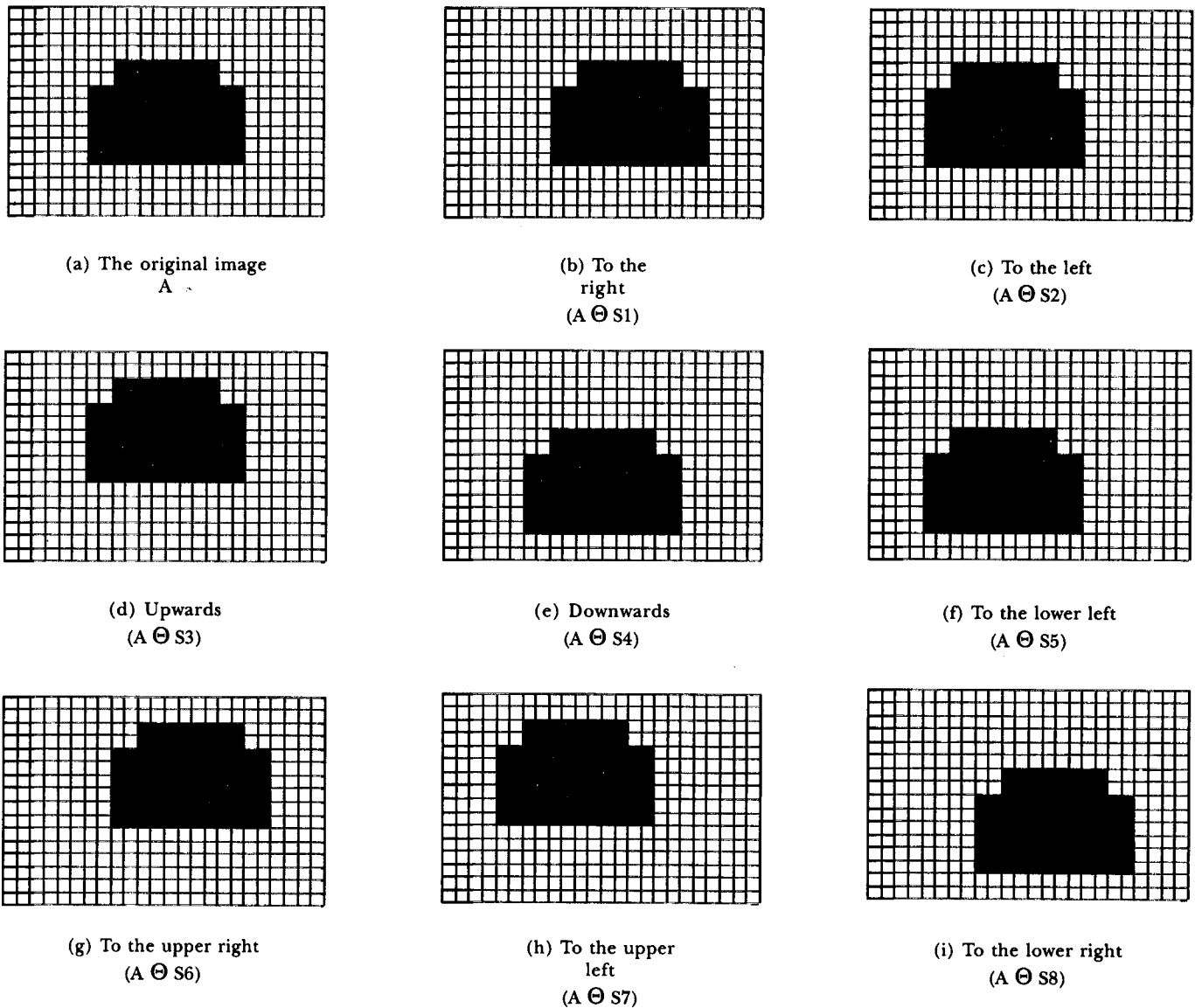
(a) The original image
A

(b) To the
right
(A ⊖ S1)

(c) To the left
(A ⊖ S2)

(d) Upwards
(A ⊖ S3)

(e) Downwards
(A ⊖ S4)

(f) To the lower left
(A ⊖ S5)

(g) To the upper right
(A ⊖ S6)

(h) To the upper
left
(A ⊖ S7)

(i) To the lower right
(A ⊖ S8)

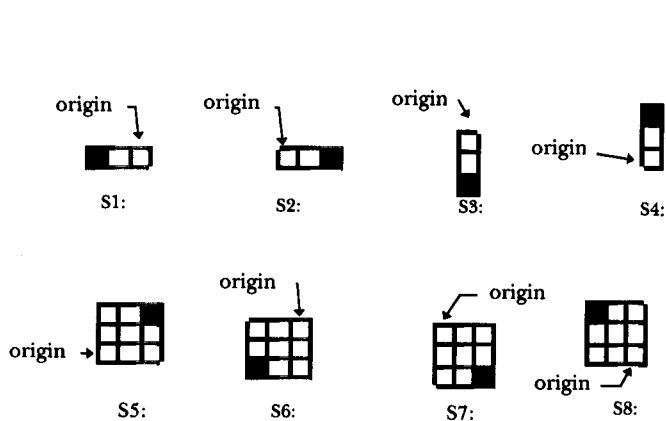*Figure 25. Feature translation by two pixels using the erosion operator.*

Figure 26. Structuring elements for translating features by two pixels that are used to produce diagrams in Figure 25.

MODIFYING A LINEAR FEATURE ADJACENT TO AN AREA FEATURE

Considering the distribution of map symbols, there are many possible neighbouring relationships, e.g., a point feature with a linear feature, two point features, a point feature with an area feature, a linear feature with an area feature, two linear features, and two area features. In some of these cases, feature displacement can be achieved by translation, as discussed in the previous section, or by erosion of the feature. In other cases, modification of the feature may be required. This section discusses feature modification in the cases of a linear feature with an area feature, and of two linear features.

The case of a linear feature with an area feature is illustrated in Figure 27, where the area feature is indicated by F and the linear feature by ABCDE. The smallest
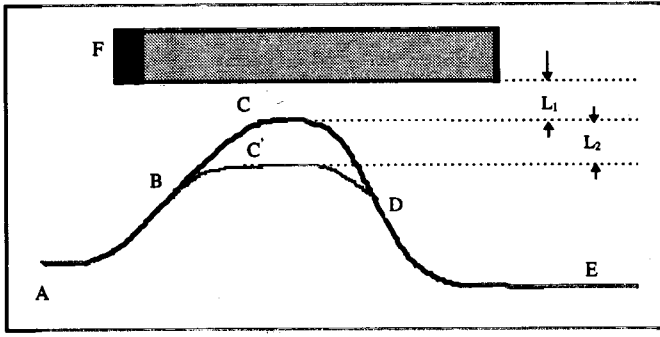
Figure 27. The linear feature to be modified in the case of a linear feature with an area feature.

distance between the area feature (F) and the linear feature (ABCDE) is L_1, which is below the threshold of separation. The objective of feature displacement is to modify the line ABCDE so that the smallest distance is increased to a higher level of $L_1 + L_2$. In this case, the BCD part of
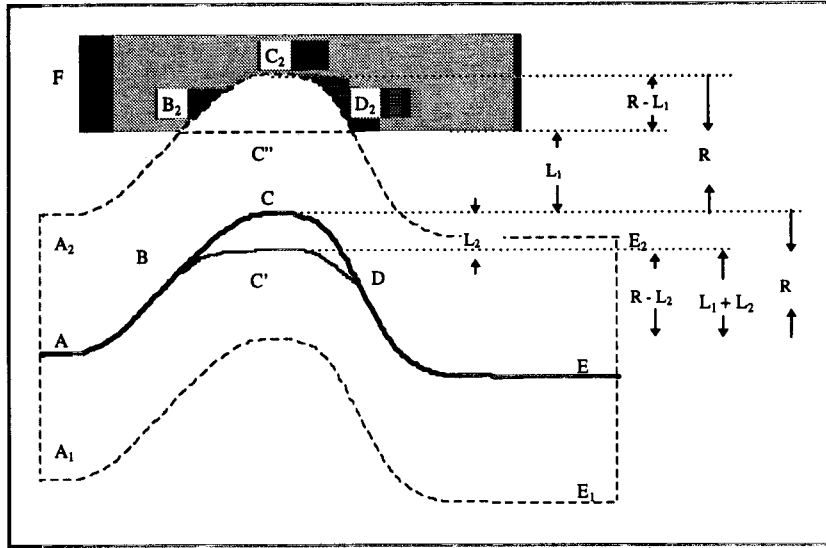

Figure 28. Modification of a linear feature.

the Line ABCDE is modified, so that it becomes BC'D.

The principle used in achieving this result with morphological operators and algorithms is illustrated in Figure 28.

The procedure is illustrated in Figure 29, and described below:

(a) Dilate the linear feature with a structuring element of size B_{size} (Figure 29 (e) and (f)), which is computed from Equations 17 and 18. In this way, the linear feature becomes an area feature, and this area feature has a cer-

tain overlap with the area feature F (Figure 29 (g)), if overlaid.

$$C = L \oplus B. \tag{11}$$

where L is the line feature and B is a structuring element.

(b) Cut the overlapping part from the dilated line obtained from Equation 11. That is, the area formed by $B_2C_2D_2$ and C" is cut so that the line becomes thinner in that part (Figure 29 (h)).

$$E = C \cap F^c \tag{12}$$

where F is the area feature that is not to be modified.

(c) Derive the skeleton of this dilated line after cutting. The skeleton of the line will be something like Line ABC'DE. That is, the linear feature is modified. Sometimes, an erosion operation is desirable in order for the skeleton to be smoother (Figure 29 (i)).

$$E_1 = E \ominus B_1 \text{ (optional)} \tag{13}$$

$$G = S(E_1) \tag{14}$$

where B_1 is another (but smaller) structuring element: $S(E_1)$ means to take the skeleton of E_1.

(d) Perform a pruning operation to remove parasitic branches (Figure 2 (j)), that will result from the skeletonization process if the dilated area is much thicker than the size of the structuring element used.

$$H = P(G) \tag{15}$$

(e) Obtain the modified result, R, by the overlay operation.

$$R = H \cup F \tag{16}$$

The size of the structuring element B (B_{size}) is determined by Equations 17 and 18. The erosion operation described in Equation 13 is optional. Experience shows that, if the size of structure element B is larger than 7 pixels, it would be better to apply the erosion process. B_1 is a structuring element whose size is determined by Equation 19.

$$R = L_1 + 2*L_2 \tag{17}$$

If expressed in terms of the number of pixels, B_{size}, then it becomes

$$B_{size} = INT\left(\frac{INT(2*R/Pixel\_size + 0.5)}{2}\right) \times 2 - 1 \tag{18}$$

$$B_{1\_size} = INT\left(\frac{INT(B_{size}/2 + 0.5)}{2}\right) \times 2 + 1 \tag{19}$$

where INT means to take the integer part of the value.

It should be noted that the pruning process is sequential, and needs a number of iterations. The actual number of it-
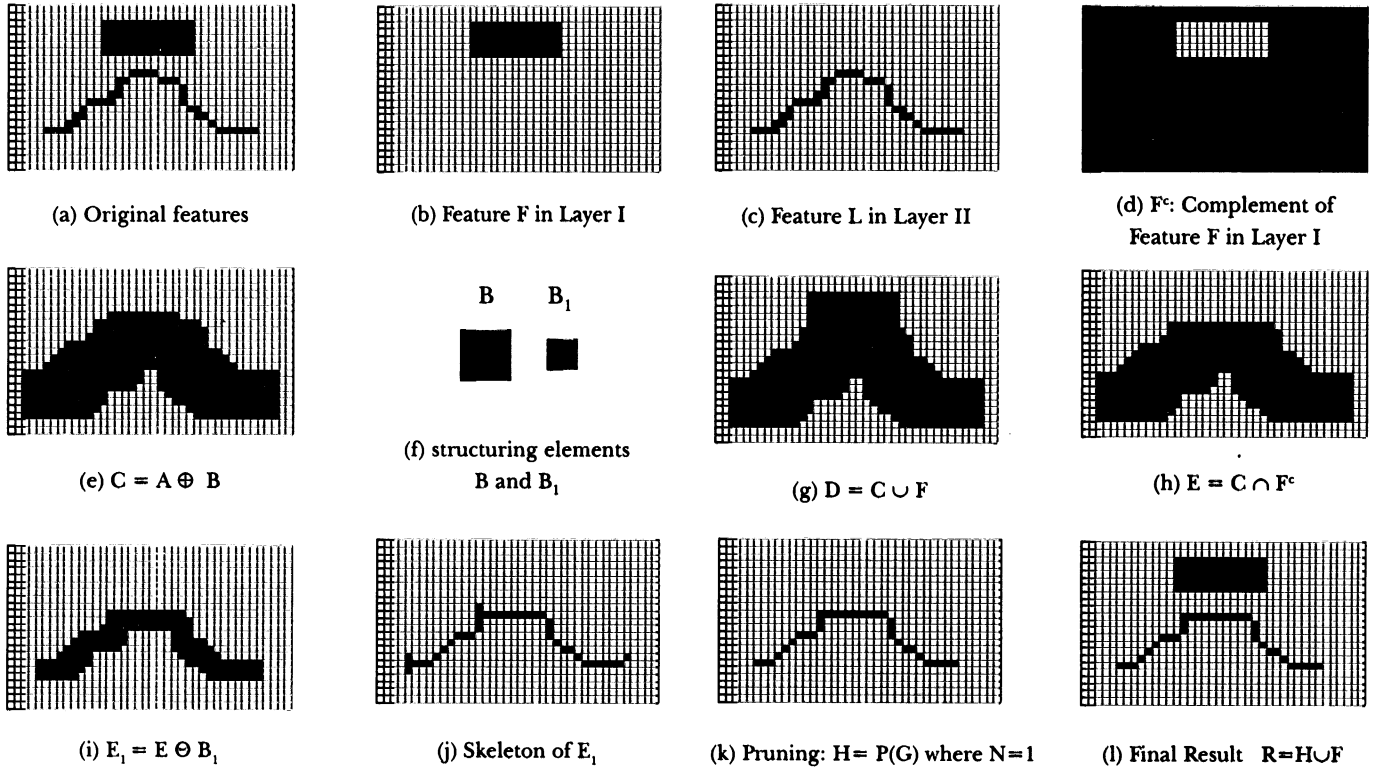
(a) Original features

(b) Feature F in Layer I

(c) Feature L in Layer II

(d) F^c: Complement of Feature F in Layer I

(e) $C = A \oplus B$

B **B$_1$**

(f) structuring elements B and B$_1$

(g) $D = C \cup F$

(h) $E = C \cap F^c$

(i) $E_1 = E \ominus B_1$

(j) Skeleton of E_1

(k) Pruning: $H = P(G)$ where $N = 1$

(l) Final Result $R = H \cup F$

Figure 29. Transformation process for modification of a linear feature

erations required, N, is dependent on the sizes of B_{size} and B_{1_size}. It is expressed as follows:

$$N = (B_{size} - B_{1\_size})/2 \quad (20).$$

However, if the optional erosion process is not applied, then it becomes:

$$N = (B_{size} - 1)/2 \quad (21).$$

MODIFICATION OF ONE FEATURE ONLY IN THE CASE OF TWO LINEAR FEATURES

In the case of two linear features, there are two possible solutions. One is to modify one feature only; the other is to modify both. This subsection will discuss the first case, and the other case will be discussed in the next section.

The first case is illustrated in Figure 30. One feature is to be modified so that the smallest distance between two linear feature is increased from L_1 to $L_1 + L_2$.

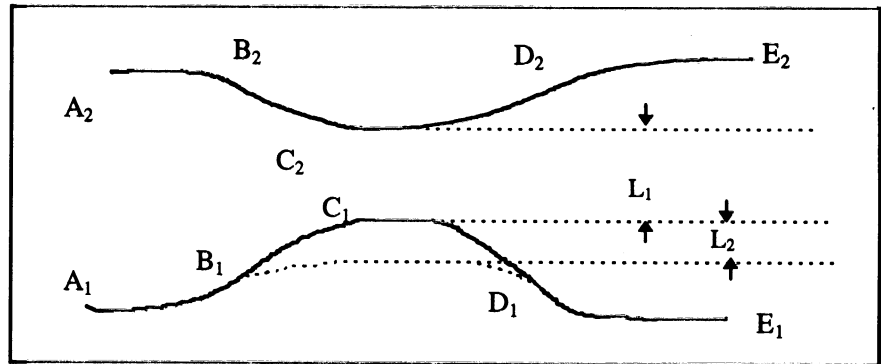The principle is very similar to that described in the previous

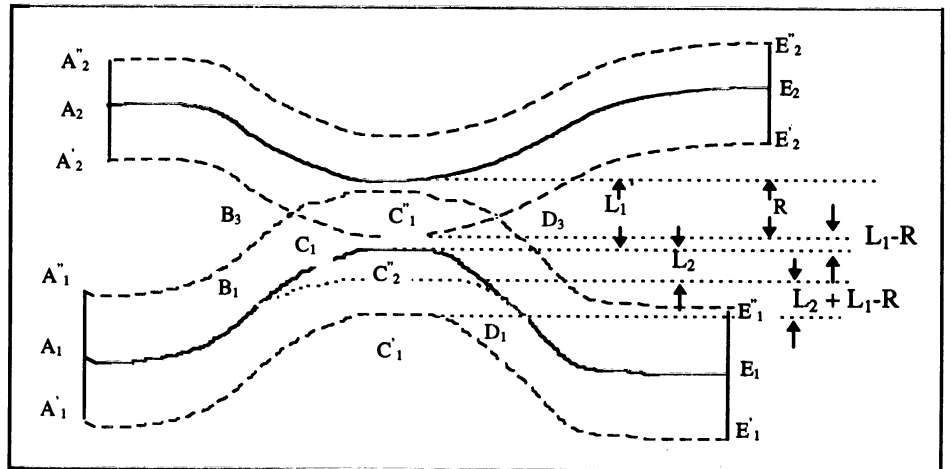*Figure 30. One of the two linear features to be modified.*

Figure 31. Modification of one of two linear features.

(a) Two original features A_1 and A_2

(b) $C_1 = A_1 \oplus B$, with $B_{size}=5$

(c) $C_2 = A_2 \oplus B$, with $B_{size}=5$

(d) $C_2 \cup C_1$

(e) $D = C_1 \cap C_2^c$

(f) $F=S(E)$ where $E=D$
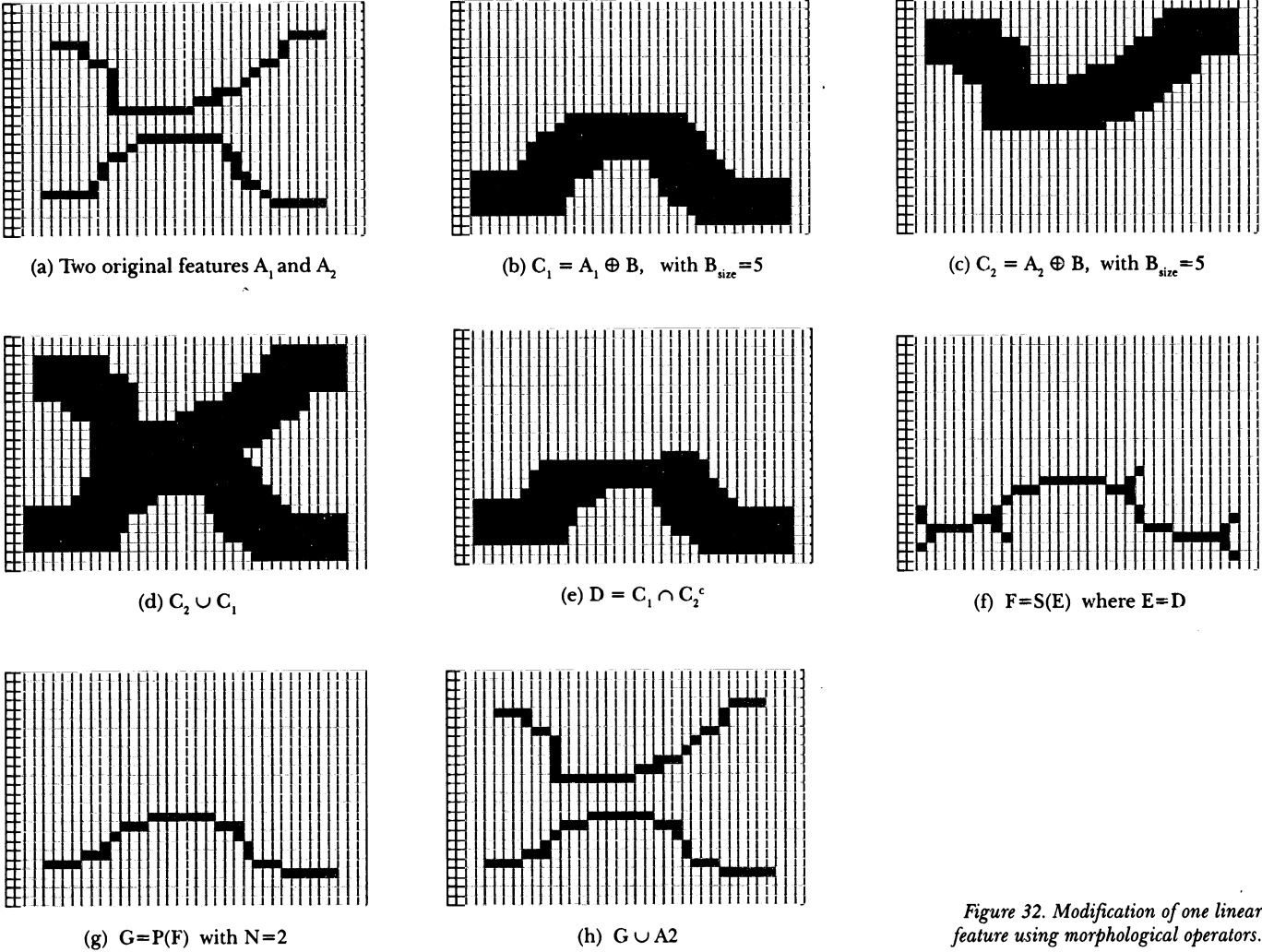
(g) $G=P(F)$ with $N=2$

(h) $G \cup A2$

Figure 32. Modification of one linear feature using morphological operators.

subsection. The only difference is that, in this case, both linear features need to be dilated in order to generate two area features with the required amount of overlap (Figure 31).

The procedure for this process is as follows:

(a) Dilate both linear features to obtain two area features:

$$C_1 = A_1 \oplus B \qquad (22),$$

$$C_2 = A_2 \oplus B \qquad (23).$$

(b) Cut off the overlapping area of the area feature dilated from the feature to be modified:

$$D = C_1 \cap C_2^c \qquad (24).$$

(c) Obtain the skeleton of the area feature that is cut:

$$E = D \ominus B_1 \text{ (optional)} \qquad (25),$$

$$F = S(E) \qquad (26).$$

(d) Perform the pruning operation to cut off parasitic branches:

$$G = P(F) \qquad (27).$$

(e) Obtain the final result by overlay operation:

$$R = G \cup A_2 \qquad (28),$$

where A_1 and A_2 are the two original linear features. Figure 32 demonstrates the transformation process from Equation 22 to Equation 28.

The size of the structuring element B is computed from Equations 29 and 30:

$$2*R=(2*L_2 + L_1) \qquad (29).$$

If it is expressed in terms of the number of pixels, B_{size}, then it becomes:

$$B_{size} = INT\left(\frac{INT(2*R/Pixel\_size + 0.5)}{2}\right) \times 2 + 1 \qquad (30),$$

where INT means to take the integer part of a value.

MODIFICATION OF BOTH FEATURES
IN THE CASE OF TWO LINEAR FEATURES

The modification of both features in the case of two linear features is illustrated in Figure 33, where $A_1B_1C_1D_1E_1$ and $A_2B_2C_2D_2E_2$ are two original linear features. The smallest distance between them is L_1, which is smaller than the threshold of separation. The modification operation is to make the distance increase to a value of $L_1 + L_2 + L_3$.

The principle is very similar to that outlined in the previous subsection, which is for the modification of one feature only. The only difference is that one needs to apply the modification process twice, once for each feature. The process is illustrated in Figure 34.
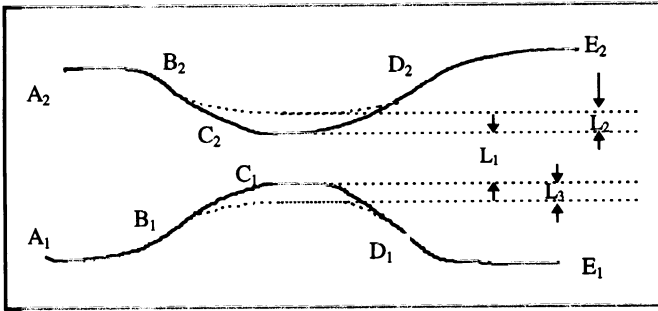

Figure 33. Both linear features to be modified.

The size of the structuring element used for the dilation of both lines, in the process of modifying the first line (e.g. Line A1 shown in Figure 33), is determined by the following formula:

$$2*R = (2*L_2 + L_1) \tag{31}.$$

If it is expressed in terms of the number of pixels, B_{size}, then it becomes:

$$B_{size} = INT\left(\frac{INT(2*R / \text{Pixel\_size} + 0.5)}{2}\right) \times 2 + 1 \tag{32},$$

where INT means to take integer part of a value.

In the process of modifying the other (second) line (e.g., Line A2, shown in Figure 33), the size of the structuring element used for the dilation of both linear features then becomes:

$$2*R = (2*L_3 + L_1 + L_2) \tag{33}.$$

If it is expressed in terms of the number of pixels, then it becomes:

$$B_{size} = INT\left(\frac{INT(2*R / \text{Pixel\_size} + 0.5)}{2}\right) \times 2 + 1 \tag{34}.$$

FEATURE MODIFICATION IN OTHER CASES

There are, of course, other cases of spatial arrangement, as was discussed at the beginning of the main section. However, the cases already discussed are very typical. The main idea in feature modification is:

(a) to dilate linear features so as to create an overlap between two features

(b) to cut the overlap area from one feature

(c) to derive the skeleton of the dilated feature, part of which has just been cut

The skeleton obtained from step (c) is the modified result one wants to have. The principles can be applied elsewhere. For example, in the case of a linear feature with a very thin area feature, one may need to dilate this area as well. As a result, the procedure described in the section on Modification of One Feature Only in the Case of Two Linear Features might be more appropriate than that described in the section on Modifying a Linear Feature in the Case of a Linear with an Area Feature.

Concluding Remarks

It has been recognized that feature displacement can be analyzed into two basic types, i.e., feature translation and modification. It is also possible to have a combination of both. This paper describes some mathematical models for both feature translation and feature modification.

For feature translation, the models are very simple. In this case, the most important thing is the set of structuring elements developed. Indeed, the key to success in morphological operators is the appropriate use of structuring elements. Using these models, features can be freely translated to any of the eight directions. The translation of features in other directions can be achieved by a combination of (some of) these eight directions. Another advantage is that all features in the same layer can be translated together, if that is desirable.

For feature modification, three special cases have been considered in detail. One is the modification of the linear feature in the case where a linear feature is close to an area feature. The other two are the modification of one linear feature only, and of both of them in the case where two lines are close together.

All the models have been tested, and results show that these models are extremely flexible and very promising.

It needs to be pointed out here that this paper aims only to provide some techniques—more precisely, mathematical models—for the displacement operation. It has been recognized by the authors that spatial conflict is the main reason for feature displacement; however, this is not the only reason, as discussed in the first section following the Introduction. That is why the detection of spatial conflicts is completely ignored in this paper.

The techniques used in this paper are in raster mode. If the original data is in vector format, then one needs to

a) rasterize the vector data;

b) perform displacement operations;

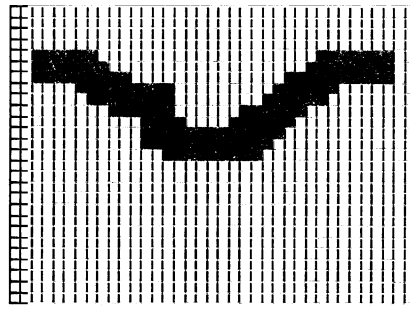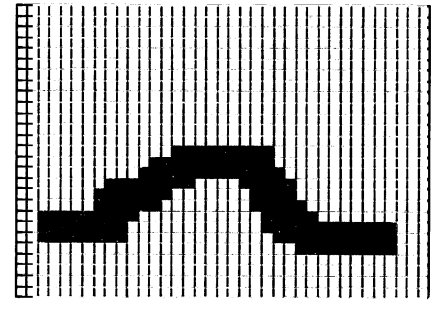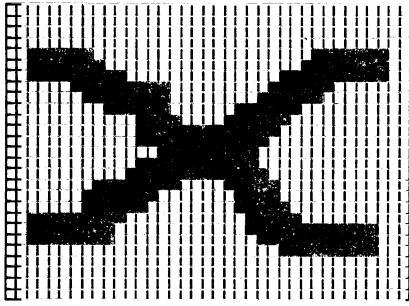c) vectorize displaced features.
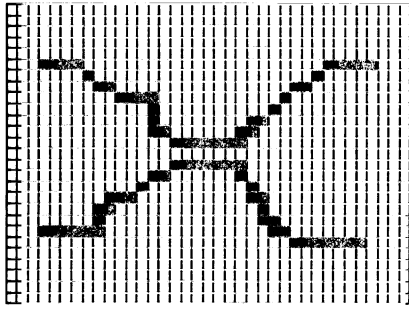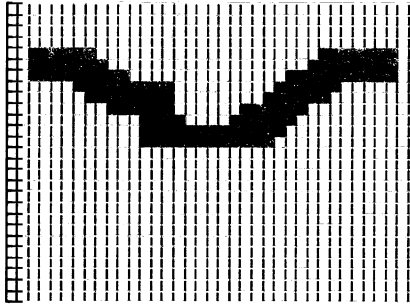
(a) Two linear features to be modified

(b) $C_1 = A_1 \oplus B$, with $B_{size} = 3$

(c) $C_2 = A_2 \oplus B$, with $B_{size} = 3$

(d) $C_1 \cup C_2$

(e) $D = C_1 \cap C_2^c$
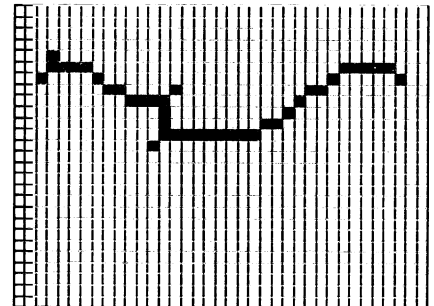
(f) $F = S(E)$ where $E = D$
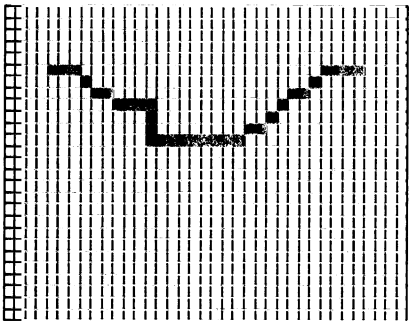
(g) $G = P(F)$ with $N = 1$
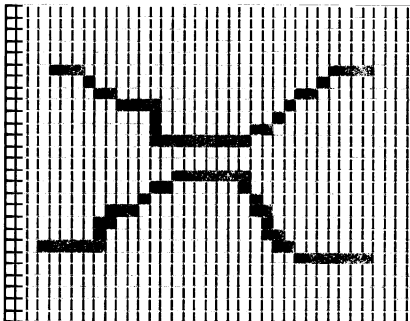
(h) $G \cup A_2$
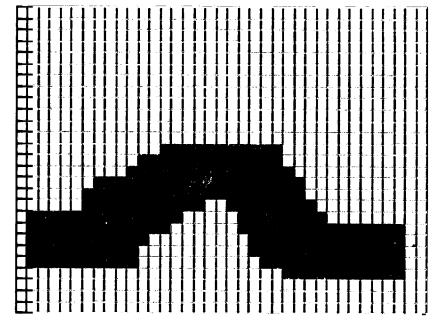
(i) $C_1 = A_2 \oplus B$ with $B_{size} = 5$
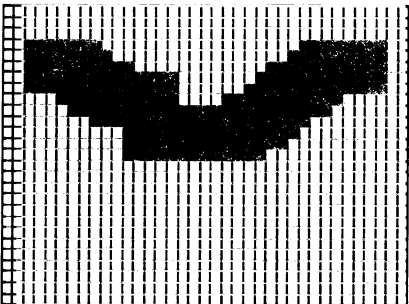
(j) $C_1 = A_1 \oplus B$ with $B_{size} = 5$ where $A_1 = G$
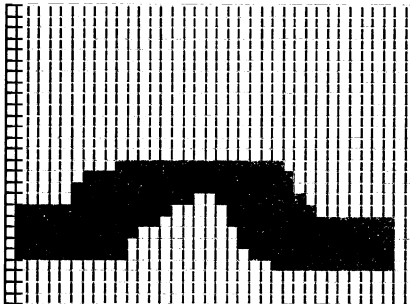
(k) $D = C_1 \cap C_2^c$
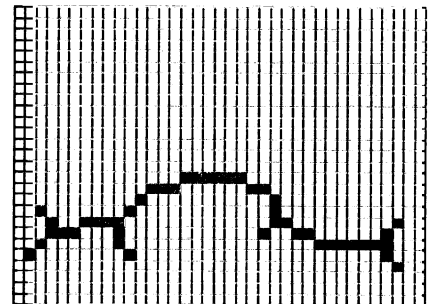
(l) $F = S(E)$ where $E = D$
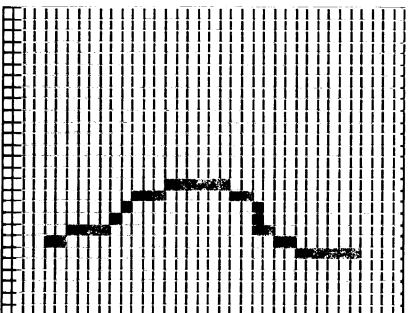
(m) $G = P(F)$ with $N = 2$
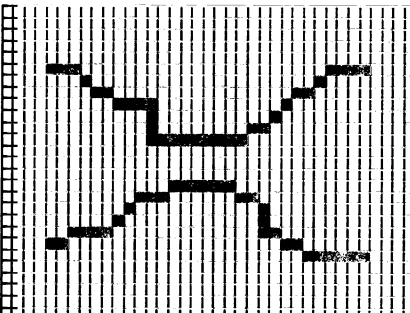
(n) $G \cap A_1$

Figure 34. Modification of both linear features using morphological operators.

It should also be pointed out here that morphological operators are good not only at feature displacement, but also at other operations in generalization, such as combination, smoothing, elimination, etc., as illustrated by Li (1994) and Su and Li (1995).

References
ABLER, R. 1987. The National Science Foundation National Center for Geographic Information and Analysis. *International Journal of Geographic Information Systems* 1/4: 303–326.

BEARD, K. AND W. MACKANESS. 1991. Generalization Operations and Supporting Structures. *Auto-Carto 10:* 29–45.

BRASSEL, K. AND R. WEIBEL. 1988. A Review and Conceptual Framework of Automated Map Generalization. *International Journal of Geographical Information Systems* 1/4: 303–326.

GOLAY, M. J. E. 1969. Hexagonal Parallel Pattern Transformation. *IEEE Transaction on Computers,* C-18, 733–740.

KEATES, J. 1989. *Cartographic Design and Production,* Second edition. London: Longman Scientific and Technical, 261 pp.

HARALICK, R., S. STERNBERG, AND X. ZHUANG. 1987. Image analysis using mathematical morphology. *IEEE Transaction of Pattern Analysis and Machine Intelligence* 9/4: 532–550.

LANTUÉJOUL, C. 1977. *Sur le modèle de Johnson-Mehl généralisé.* Fontainebleau, France: Rapport Interne Centre de Morph. Math.

LI, D. AND X-Y CHEN. 1991. Automatically Generating Triangulated Irregular Digital Terrain Model Networks by Mathematical Morphology. *ISPRS Journal of Photogrammetry and Remote Sensing* 46: 283–295.

LI, Z. 1994. Mathematical Morphology in Digital Generalization of Raster Map Data. *Cartography* 23/1: 1–10.

LI, Z. AND S. OPENSHAW. 1992. Algorithms for Automated Line Generalization Based on a Natural Principle of Objective Generalization. *International Journal of Geographical Information Systems* 6/5: 373–389.

——. 1993. A Natural Principle for the Objective Generalization of Digital Maps. *Cartography and Geographic Information Systems* 20/1: 19–29.

MARBLE, D. 1984. Geographic Information Systems: An Overview. *Proceedings of Pecora 9,* Sioux Falls, SD: 18–24. Reprinted in *Introductory Readings in Geographic Information Systems,* D. J. Peuquet and D. F. Marble, ed. Taylor & Francis, London. 8–17.

MATHERON, G. 1975. *Random Sets and Integral Geometry.* New York: Wiley.

MEYER, F. 1979, *Cytologie quantitative et Morphologie mathématique,* Thèse docteur ingénieur, École des Mines de Paris.

MONMONIER, M. 1987. Displacement in Vector- and Raster-mode Graphics. *Cartographica* 24/4: 25–36.

MÜLLER, J.-C. 1989. Theoretical Considerations for Automated Map Generalization. *ITC Journal* 3/4: 200–204.

——. 1991. The Cartographic Agenda of the 1990s: Updates and Prospects. *ITC Journal* 1992/2: 55–62.

MÜLLER, J.-C., R. WEIBEL, J. P LAGRANGE, AND F. SALGE. 1993. Generalization: State of the Art and Issues. *GISDATA Task Force on Generalization 1993.* European Science Foundation. 21 pp. Also in *GIS and Generalization,* J.-C. Müller, J. P. Lagrange, and R. Weibel, eds. London: Taylor & Francis, 1995): 3–17.

RHIND, D. 1973. Generalization and Realism within Automated Cartographic Systems. *Canadian Cartographer* 10/1: 51–62.

——. 1988. A GIS Research Agenda. *International Journal of Geographical Information Systems* 2/1: 23–28.

ROBINSON, A., R. SALE, AND J. MORRISON. 1978. *Elements of Cartography,* 4th ed. New York: John Wiley & Sons, 448 pp.

ROBINSON, G. 1993. Generalization: State of the Art and Issues — A Response. *A report to the European Science Foundation.* Reading, UK: University of Reading, 10 pp.

SERRA, J. 1982. *Image Processing and Mathematical Morphology.* New York: Academic Press, 610 pp.

SHEA, K. AND R. McMASTER. 1989. Cartographic Generalization in a Digital Environment: When and How to Generalize. *Proceedings of Auto-Carto 9,* 56–67.

SU, B. AND Z. LI. 1995. An Algebraic Basis for Digital Generalization of Area-patches Based on Morphological Techniques. *The Cartographic Journal* 32/2: 148–153.

WEIBEL, R. 1995. *Summary Report: Workshop on Progress in Automated Map Generalization.* Technical Report. ICA Working Group on Automated Map Generalization, Barcelona.

Résumé   Modèles algébriques pour le déplacement d'éléments lors de la généralisation de données cartographiques numériques au moyen de techniques morphologiques   Le déplacement d'éléments est une des nombreuses opérations (ou opérateurs) de la généralisation de données cartographiques. C'est même devenu un sujet prioritaire dans le monde de la recherche. Cet article décrit quelques modèles algébriques (ou modèles mathématiques) pour cette opération. Ces modèles sont basés sur les opérateurs développés en morphologie mathématique, science qui traite de la forme et de la structure des objets.

On peut classifier les déplacements d'éléments en deux groupes : la translation et la modification d'élément. Dans le premier cas, on développe un ensemble d'éléments structurants, éléments clés des opérateurs morphologiques. En utilisant cet ensemble d'éléments structurants, on peut employer les deux opérateurs morphologiques de base, soit la dilatation et l'érosion, pour déplacer librement des éléments dans chacune des huit directions. On peut aussi déplacer des éléments dans d'autres directions en combinant ces huit directions. En ce qui concerne le deuxième type de déplacement d'élément, plusieurs modèles ont été développés pour

résoudre les différents cas suivants : un élément de surface avec un élément linéaire, deux éléments linéaires, etc. On a aussi testé ces modèles en utilisant de nombreux exemples. Les résultats montrent que ces techniques sont très prometteuses.