Mathematical Morphology in Digital Generalization of Raster Map Data

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This paper discusses the potential application of mathematical morphology techniques in digital generalization of raster map data. A review of existing generalization operators and morphological operators is conducted so that an insight into their relationships can then be made. The potential applications of morphological tools in digital map generalization of raster map data is discussed and problems associated with such an application are also identified. In particular, four application areas–elimination, combination, line simplification and displacement–are successfully demonstrated and the results show that morphological operators could be very important tools for generalisation purposes.

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Introduction

Generalization is a traditional topic in cartography and has become an important function of a geographical information system (GIS). As Marble (1984) points out, "The need to aggregate and generalize spatial data is a continuing technical problem in spatial data handling, and it becomes more severe as databases grow larger and approach global coverage". However, unfortunately, "(sound) generalization procedures to transform the information content of a map from one scale to another are notably absent within the realm of functions usually available in GIS" (Muller, 1989). "It is not surprising that the issue of generalization is included in the international GIS research agenda (Abler, 1987; Rhind, 1988)" (Li and Openshaw, 1993). The importance of this topic has been highlighted by the fact that it has become one of the 12 NCGIA (National Center for Geographic Information and Analysis, USA) initiatives under the title of "multi representation". Indeed, this topic has also become the initiative of other research institutions around the world, such as the Regional Research Laboratorys (RRL) in Britain—UK's equivalent to US NCGIA.

So far, most research efforts have been spent on the generalization of vector data, particularly vector linear features. Only very few papers (e.g. Monmonier, 1983) have dealt with raster data. However, it should be more convenient to carry out generalization operations in raster mode, since generalization is caused by a reduction in map space where map scale is reduced, and raster is a space-primary data structure. Therefore, in this study, attention is paid to generalization of raster map data.

For raster data processing, one may automatically think of the well-established techniques of digital image processing and try to borrow some ideas or tools from there. Indeed, it is an attempt of this study to investigate the feasibility of borrowing morphological tools (operators) from digital image processing for generalization purposes.

In this paper, existing generalization operators are first of all examined; then follows a review of morphological tools (operators); next the feasibility of applying morphological tools to digital generalization of raster map data is investigated, and finally problems associated with such an application are identified.

Generalization operators

First of all, the definition of generalization is briefly discussed so that generalization operators can be introduced smoothly.

Generalization is a concept which is difficult to define, partly because its contents have not yet been fully understood. Therefore, existing definitions are rather vague. Robinson et al (1978) defined it as a modification process. They state: "There are a variety of modifications that can, and must, be carried out as a result of reduction; they range from essential mechanical processes to intellectual exercises. These modifications collectively are called cartographic generalization." Similarly, Keates (1989) describes generalization as an adjustment process. He states: "As a map is always at a smaller scale than the phenomena it represents, the information it contains must be restricted by what can be presented graphically at map scale. This process is referred to as generalization". The important issue arising is how such a modification or an adjustment process can be carried out, especially by computer. It means that some kind of operations (or operators) need to be identified.

Indeed, various sets of generalization operators have been identified by researchers and some of them are listed in Table 1. It can be seen that some of them are similar and others are different. It needs to be noted here that the same terminology may mean different things in different sets of operators. For example, Keates' simplification is very different from Shea and McMaster's simplification. In the former, simplification means that unwanted small details (including spikes) in line features are removed with a smoothing result. However, in the latter, simplification means removing some points from the line feature. Keates' simplification may be better referred to as line generalization, as suggested by Li and Openshaw (1992). In order to avoid confusion, the author will indicate whose terminology is used where referred to in this paper.

Proposers	Set of Operators
Keates (1989)	Selective omission, Simplification, Combination, Exaggeration, Displacement
Robinson et al (1978)	Simplification, Classfication, Symbolisation, Induction
Rhind (1973)	Line sinuosity reduction, Feature transportation, Amalgamation, Elimination, Graphic coding
Beard and Mackaness (1991)	Selection, Omission, Coarsening, Collapsing, Combination, Classification, Exaggeration, Displacement
Shea and McMaster (1989)	Simplication, Smoothing, Aggregation, Amalgamation, Merging, Collapse, Refinement, Typification, Exaggeration, Enhancement, Displacement, Classification

Table 1. *Generalization Operators*

Morphological operators

In the previous section, generalization operators were reviewed. Morphological operators are now introduced so that a comparison of both can be made.

Mathematical morphology is a science of form and structure, based on set theory. It was developed by French geostatistical scientists G. Matheron and J. Serra in 1965 (Matheron, 1965; Serra, 1982). It has found increasing application in digital image processing. Efforts have also been made by researchers on applying morphological tools to mapping related sciences, such as in digital terrain modelling (Li and Chen, 1989). In order to have an intuitive understanding of the potential application of morphological tools in map generalization, these tools (operators) will now be examined, illustrated and related to generalization operators.

The basic morphological operators are dilation and erosion. They are defined as follows (see Serra, 1982; Haralick *et al*, 1987):

Dilation: $A \oplus B = \{a + b: a \in A, b \in B\} = \cup_{b \in B} A_b$ (1)

Erosion: $A \ominus B = \{a: a + b \in A, b \in B\} = \cap_{b \in B} A_b$ (2)

where A is the image to be processed and B is called the structure element, which can be considered to be an analogy to the kernel in convolution operations and is usually a 2x2 or 3x3 image. In Eq.(1), it is called "dilation of A by B" and in Eq.(2) "erosion of A by B". Examples of these two operators are given in Fig.1, where a 3x3 image is used for the structure element. In these diagrams, "+"

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0
0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0
0	0	0	1	1	1	1	1	1	0	0	0	0	1	0	0
0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0
0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	0	1	0	0	0
0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0
0	0	1	0	0	0	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0
0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(a) Original image A

1	1	1
1	(1)	1
1	1	1

(b) Structure element B

0	0	0	0	0	+	+	+	+	+	0	0	0	0	0	0
0	0	0	0	0	+	1	1	1	+	+	+	0	0	0	0
0	0	0	+	+	+	1	1	1	1	1	+	0	0	0	0
0	0	+	+	1	1	1	1	1	1	1	+	0	0	0	0
0	0	+	1	1	1	1	1	1	1	+	+	+	+	+	0
0	0	+	1	1	1	1	1	1	+	+	0	+	1	+	0
0	0	+	+	1	1	1	1	1	+	+	+	+	+	+	0
0	0	+	+	1	1	1	1	1	1	1	+	0	0	0	0
0	0	+	1	1	1	1	1	+	+	+	+	0	0	0	0
0	0	+	+	1	+	+	+	+	1	+	+	+	+	0	0
0	0	0	+	1	1	1	1	1	1	1	+	1	+	0	0
0	+	+	+	+	+	1	1	1	1	1	1	+	+	0	0
0	+	1	+	0	+	1	1	1	1	1	1	+	+	+	0
0	+	+	+	+	+	1	1	1	1	1	1	1	1	+	0
0	0	0	+	1	+	1	+	+	+	+	+	+	+	+	0
0	0	0	+	+	+	+	+	0	0	0	0	0	0	0	0

(c) Dilation of A by B
("+" pixels are expanded)

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	–	–	–	0	0	0	0	0	0	0
0	0	0	0	0	0	–	1	–	–	–	0	0	0	0	0
0	0	0	0	–	–	–	1	1	–	–	0	0	0	0	0
0	0	0	–	–	1	1	1	–	–	0	0	0	0	0	0
0	0	0	–	–	1	1	1	–	0	0	0	0	–	0	0
0	0	0	0	–	1	1	1	–	0	0	0	0	0	0	0
0	0	0	0	–	1	1	–	–	–	–	0	0	0	0	0
0	0	0	–	–	–	–	–	0	0	0	0	0	0	0	0
0	0	0	0	–	0	0	0	0	–	0	0	0	0	0	0
0	0	0	0	–	–	–	–	–	–	–	0	–	0	0	0
0	0	0	0	0	0	–	1	1	1	–	–	0	0	0	0
0	0	–	0	0	0	–	1	1	1	1	–	0	0	0	0
0	0	0	0	0	0	–	–	–	–	–	–	–	–	0	0
0	0	0	0	–	0	–	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(d) Erosion of A by B
("–" pixels are eroded)

Figure 1. *The effect of dilation and erosion on the shape of an image.*

represents the pixel which becomes "1" after dilation and "-" represents the pixel becoming "0" after erosion. It illustrates clearly how these two operators affect the shape of the original image. Of course, the structure element is a critical element in these operations, as will be seen later.

An idea similar to these two operators, i.e. Perkal's blanket method, has been applied to generalisation (see Muller, 1991; Muller and Wang, 1992). However, that idea is not exactly the same as the two operators discussed above and it has no established algebraic basis at all. There are other similar ideas used in map generalisation such as those used by Monmonier (1993), however, they are different from mathematical morphology.

Based on these two basic operators, a number of new operators have also been developed. Some of them are relevant to map generalization and also listed as follows:

Opening: $A \circ B = (A \ominus B) \oplus B$ (3)

Closing: $A \bullet B = (A \oplus B) \ominus B$ (4)

Indeed, opening is a process with two stages. The first is erosion, followed by dilation. The closing is also a two stage process but in reverse order. The effect of these two operators on the shape of the original image is illustrated in Fig.2. Fig.2(c) shows how the closing operator works, i.e. the image of A dilated by B, then eroded by B. Fig.2(d) shows how the opening operator works, i.e. the eroded image dilated by B.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0
0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0
0	0	0	1	1	1	1	1	1	0	0	0	0	1	0	0
0	0	0	0	1	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0
0	0	0	1	1	1	1	1	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	0	1	0	0	0
0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0
0	0	1	0	0	0	1	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0
0	0	0	0	1	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(a) Original image A

1	1	1
1	(1)	1
1	1	1

(b) Structure element B

0	0	0	0	0	−	−	−	−	−	0	0	0	0	0	0
0	0	0	0	0	−	1	1	1	−	−	−	0	0	0	0
0	0	0	−	−	−	1	1	1	1	1	−	0	0	0	0
0	0	−	−	1	1	1	1	1	1	1	−	0	0	0	0
0	0	−	1	1	1	1	1	1	1	−	−	−	−	−	0
0	0	−	1	1	1	1	1	1	+	−	0	−	1	−	0
0	0	−	+	1	1	1	1	1	+	−	−	−	−	−	0
0	0	−	+	1	1	1	1	1	1	1	−	0	0	0	0
0	0	−	1	1	1	1	1	+	+	+	−	0	0	0	0
0	0	−	−	1	+	+	+	+	1	+	−	−	−	0	0
0	0	0	−	1	1	1	1	1	1	1	+	1	−	0	0
0	−	−	−	−	−	1	1	1	1	1	1	+	−	0	0
0	−	1	−	0	−	1	1	1	1	1	1	+	−	−	0
0	−	−	−	−	−	1	1	1	1	1	1	1	1	−	0
0	0	0	−	1	+	1	−	−	−	−	−	−	−	−	0
0	0	0	−	−	−	−	−	0	0	0	0	0	0	0	0

(c) Closing of A by B
(1st dilation, 2nd erosion
("+"=1, "−"=0)

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	+	+	+	0	0	0	0	0	0	0
0	0	0	0	0	0	+	1	+	+	−	0	0	0	0	0
0	0	0	0	+	+	+	1	1	+	−	0	0	0	0	0
0	0	0	−	+	1	1	1	+	+	0	0	0	0	0	0
0	0	0	−	+	1	1	1	+	0	0	0	0	−	0	0
0	0	0	0	+	1	1	1	+	0	0	0	0	0	0	0
0	0	0	0	+	1	1	+	+	−	−	0	0	0	0	0
0	0	0	−	+	+	+	+	0	0	0	0	0	0	0	0
0	0	0	0	−	0	0	0	0	−	0	0	0	0	0	0
0	0	0	0	−	−	+	+	+	+	+	0	−	0	0	0
0	0	0	0	0	0	+	1	1	1	+	+	0	0	0	0
0	0	−	0	0	0	+	1	1	1	1	+	0	0	0	0
0	0	0	0	0	0	+	+	+	+	+	+	−	−	0	0
0	0	0	0	−	0	−	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(d) Opening of A by B
(1st erosion, 2nd dilation
("+"=1, "−"=0)

Figure 2. *The effect of closing and opening on the shape of an image.*

There are also many other operators such as thinning, thickening, hit or miss, conditional dilation, conditional erosion, conditional thinning, conditional thickening, sequential dilation, and conditional sequential dilation, and so on. However, it is not the purpose of this paper to discuss all of them.

Potential application of morphological tools in map generalization

By now, it should be clear that morphological operators are very powerful tools for manipulation of raster map data. In this section, an attempt is made to connect these operators to some of the generalization operators, i.e. to discuss the potential applications in map generalization. In particular, four areas commonly applied in map generalisation will be considered, i.e. combination, elimination, simplification and displacement.

First of all, the *combination* operator (Keates; 1989; Beard and Mackaness, 1991) will be considered. It is illustrated in Fig.3. Fig.3(a) is the original map, where there are four clusters to be combined. Fig.3(b) is the result of the original map dilated by structure element B (in Fig.1 and and Fig.2), where the "+" means that this pixel becomes "1" as a result of this dilation operation. It can be seen that there are still two "0"s left in the map. They are to be removed by another dilation operation and the result is shown in Fig.3(c), where "#" means that this pixel becomes "1" as a result of the second dilation process. Alternatively, if a larger structure element were used, one dilation operation might be sufficient.

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0
0 0 0 0 1 1 1 1 1 1 0 0 0 0 1 1 1 1 1 0 0 0
0 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 1 1 1 1 0 0
0 0 1 1 1 1 1 1 1 0 1 1 1 1 0 0 0 1 1 1 0 0
0 0 0 1 1 1 1 0 0 0 0 1 1 1 0 0 1 1 1 0 0 0
0 0 0 0 1 1 1 1 0 0 1 1 1 1 1 0 1 1 1 1 0 0
0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 1 1 1 0 0 0 0
0 0 0 1 1 1 1 0 0 0 0 0 0 0 0 0 1 1 1 1 0 0
0 0 1 1 1 1 1 0 0 0 1 1 1 0 0 0 0 1 1 1 0 0
0 0 0 1 1 1 0 0 0 0 1 1 1 1 1 0 1 1 1 0 0 0
0 0 0 0 1 1 1 1 0 0 0 0 1 1 1 0 0 1 1 1 0 0
0 0 0 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

(a) Original image C

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 + + + + + + + 0 0 0 0
0 0 0 + + + + + + + + + 1 1 1 1 1 + + + 0 0
0 0 + + 1 1 1 1 1 1 + + + + 1 1 1 1 1 + + 0
0 + + 1 1 1 1 1 1 + + + + + + + 1 1 1 1 + 0
0 + 1 1 1 1 1 1 1 + 1 1 1 1 + + + 1 1 1 + 0
0 + + 1 1 1 1 + + + + 1 1 1 + + 1 1 1 + + 0
0 0 + + 1 1 1 1 + + 1 1 1 1 1 + 1 1 1 1 + 0
0 0 + + + 1 1 + + + + + + + + 1 1 1 + + + 0
0 + + 1 1 1 1 + 0 + + + + + + + 1 1 1 1 + 0
0 + 1 1 1 1 1 + 0 + 1 1 1 + + + + 1 1 1 + 0
0 + + 1 1 1 + + + + 1 1 1 1 1 + 1 1 1 + + 0
0 0 + + 1 1 1 1 + + + + 1 1 1 + + 1 1 1 + 0
0 0 0 + + 1 1 1 1 1 1 + + + + + + + + + + 0
0 0 0 0 + + + + + + + + 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

(b) C⊕B

```
0 0 0 0 0 0 0 0 0 0 # # # # # # # # # 0 0 0
0 0 # # # # # # # # # + + + + + + + # # # 0
0 # # + + + + + + + + + 1 1 1 1 1 + + + # #
# # + + 1 1 1 1 1 1 + + + + 1 1 1 1 1 + + #
# + + 1 1 1 1 1 1 + + + + + + + 1 1 1 1 + #
# + 1 1 1 1 1 1 1 + 1 1 1 1 + + + 1 1 1 + #
# + + 1 1 1 1 + + + + 1 1 1 + + 1 1 1 + + #
# # + + 1 1 1 1 + + 1 1 1 1 1 + 1 1 1 1 + #
# # + + + 1 1 + + + + + + + + 1 1 1 + + + #
# + + 1 1 1 1 + # + + + + + + + 1 1 1 1 + #
# + 1 1 1 1 1 + # + 1 1 1 + + + + 1 1 1 + #
# + + 1 1 1 + + + + 1 1 1 1 1 + 1 1 1 + + #
# # + + 1 1 1 1 + + + + 1 1 1 + + 1 1 1 + #
0 # # + + 1 1 1 1 1 1 + + + + + + + + + + #
0 0 # # + + + + + + + + # # # # # # # # # #
0 0 0 # # # # # # # # # # 0 0 0 0 0 0 0 0 0
```

(c) C⊕B⊕B

```
0 0 0 0 0 0 0 0 0 0 - - - - - - - - - 0 0 0
0 0 - - - - - - - - - + + + + + + + - - - 0
0 0 - - + + + + + + + + 1 1 1 1 1 + + + - -
- - + + 1 1 1 1 1 1 + + + + 1 1 1 1 1 + + -
- + + 1 1 1 1 1 1 + + + + + + + 1 1 1 1 + -
- + 1 1 1 1 1 1 1 + 1 1 1 1 + + + 1 1 1 + -
- + + 1 1 1 1 + + + + 1 1 1 + + 1 1 1 + + -
- # + + 1 1 1 1 + + 1 1 1 1 1 + 1 1 1 1 + -
- # + + + 1 1 + + + + + + + + 1 1 1 + + + -
- + + 1 1 1 1 + # + + + + + + + 1 1 1 1 + -
- + 1 1 1 1 1 + # + 1 1 1 + + + + 1 1 1 + -
- + + 1 1 1 + + + + 1 1 1 1 1 + 1 1 1 + + -
- - + + 1 1 1 1 + + + + 1 1 1 + + 1 1 1 + -
0 - - + + 1 1 1 1 1 1 + + + + + + + + + + -
0 0 - - + + + + + + + + - - - - - - - - - -
0 0 0 - - - - - - - - - - 0 0 0 0 0 0 0 0 0
```

(d) C⊕B⊕B⊖B

Figure 3. *Combination operation using morphological operators.*

0	0	0	0	0	0	0	0	0	0	-	-	-	-	-	-	-	-	-	0	0	0
0	0	-	-	-	-	-	-	-	-	-	=	=	=	=	=	=	=	-	-	-	0
0	0	-	-	=	=	=	=	=	=	=	=	1	1	1	1	1	=	=	=	-	-
-	-	=	=	1	1	1	1	1	1	+	+	+	+	1	1	1	1	1	=	=	-
-	=	=	1	1	1	1	1	1	+	+	+	+	+	+	+	1	1	1	1	=	-
-	=	1	1	1	1	1	1	1	+	1	1	1	1	+	+	+	1	1	1	=	-
-	=	+	1	1	1	1	+	+	+	+	1	1	1	+	+	1	1	1	+	=	-
-	=	+	+	1	1	1	1	+	+	1	1	1	1	1	+	1	1	1	1	=	-
-	=	+	+	+	1	1	+	+	+	+	+	+	+	+	1	1	1	+	+	=	-
-	=	+	1	1	1	1	+	#	+	+	+	+	+	+	+	1	1	1	1	=	-
-	=	1	1	1	1	1	+	#	+	1	1	1	+	+	+	+	1	1	1	=	-
-	=	=	1	1	1	+	+	+	+	1	1	1	1	1	+	1	1	1	+	=	-
-	-	=	=	1	1	1	1	+	+	+	+	1	1	1	+	+	1	1	1	=	-
0	-	-	=	=	1	1	1	1	1	1	=	=	=	=	=	=	=	=	=	=	-
0	0	-	-	=	=	=	=	=	=	=	=	-	-	-	-	-	-	-	-	-	-
0	0	0	-	-	-	-	-	-	-	-	-	-	0	0	0	0	0	0	0	0	0

(e) C⊕B⊕B⊖B⊖B

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0
0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(f) Final Image

Figure 3 (Continued). *Combination operation using morphological operators.*

So far, the map is expanded and should be shrunken back by applying the erosion operation twice, with the same structure element B. Fig.3(d) shows the result after applying one erosion operation, where "-" means that this pixel becomes "0" as a result of this erosion operation. Fig.3(e) shows the result after the second erosion operation, where "=" means that this pixel becomes "0" as a results of the second erosion operation. The final map is shown in Fig.3(f). It should be noted here that "+", "=", "-" and "#" will have the same meanings throughout this paper.

Now turn to the *elimination* (Keates, 1989) or coarsening (Beard and Mackaness, 1991) operator. It is illustrated in Fig.4. Fig.4(a) is the original image, where there are three small clusters to be eliminated. By applying erosion with the structure element B (Fig.1 and Fig.2), there are only three "1" pixels left, as shown in Fig.4(b). After applying the second erosion operation, all "1"s in these three clusters are removed. Again, if a larger structure element were used, one erosion operation might be sufficient.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0
0	0	0	0	0	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(a) Original image D

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-	-	-	0	0	0	0	-	-	-	-	0	0	0	0	0	0
0	0	0	0	0	-	1	-	0	0	0	0	-	1	1	-	0	0	0	0	0	0
0	0	0	0	0	-	-	-	0	0	0	0	-	-	-	-	0	0	0	0	0	0
0	0	0	0	0	0	-	0	0	0	0	0	0	-	-	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-	-	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-	-	-	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(b) D⊖B

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	-	-	-	0	0	0	0	-	-	-	-	0	0	0	0	0	0
0	0	0	0	0	-	=	-	0	0	0	0	-	=	=	-	0	0	0	0	0	0
0	0	0	0	0	-	-	-	0	0	0	0	-	-	-	-	0	0	0	0	0	0
0	0	0	0	0	0	-	0	0	0	0	0	0	-	-	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-	-	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	-	-	-	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(c) D⊖B⊖B

Figure 4. *Elimation (coarsening) operation using morphological operators*

Next, the *simplification* (Keates, 1989) of boundary lines will be considered. This is illustrated in Fig.5. Fig.5(a) is the original map, whose boundary has high sinuosity. Fig.5(b) is the structure element F to be used. By applying erosion and dilation in some combination, as shown in Fig.5(c), and (d), the boundary line is simplified and the final result is shown in Fig.5(f).

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 1 0 0 0 0 0
0 0 0 0 1 1 1 0 0 0 1 1 1 1 1 1 1 1 1 0 0 0
0 0 0 0 0 1 1 1 0 0 1 1 1 1 1 1 0 1 1 1 0 0
0 0 1 0 0 0 1 1 1 0 0 1 1 1 1 0 0 0 0 0 0 0
0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0
0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0 1 0 0 0
0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
0 0 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1 1 1 0 0 0
0 0 0 1 1 1 1 1 1 0 0 1 1 1 1 1 1 1 1 1 0 0
0 0 0 0 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1 0 0 0
0 0 0 0 0 1 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

(a) Original image E

```
* 1 *
1 ① 1
* 1 *
```

* Means not defined

(b) Structure element F

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 - - - - - 0 0 0 0 0
0 0 0 0 - - - 0 0 0 - - 1 1 1 1 - - - 0 0 0
0 0 0 0 0 - 1 - 0 0 - 1 1 1 1 - 0 - - - 0 0
0 0 - 0 0 0 - 1 - 0 0 - 1 1 - 0 0 0 0 0 0 0
0 0 - - - - 1 1 1 - - 1 1 1 1 - - 0 0 0 0 0
0 0 - - 1 1 1 1 1 1 1 1 1 1 1 - 0 0 - 0 0 0
0 0 0 0 - 1 1 1 1 1 1 1 1 1 1 1 - - 1 - 0 0
0 0 0 0 - 1 1 - - - 1 1 1 1 1 1 1 1 1 - 0 0
0 0 - - 1 1 - 0 0 0 - 1 1 1 1 1 1 1 - 0 0 0
0 0 0 - 1 1 1 - - 0 0 - 1 1 1 1 1 1 1 - 0 0
0 0 0 0 - 1 1 1 - 0 0 0 - - - - - - - 0 0 0
0 0 0 0 0 - - - - - 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

(c) $E \ominus F$

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 + + + + - 0 0 0 0 0
0 0 0 0 - - + 0 0 0 - + 1 1 1 1 + - - 0 0 0
0 0 0 0 0 + 1 + 0 0 + 1 1 1 1 + 0 - - - 0 0
0 0 - 0 0 0 + 1 + 0 0 + 1 1 + 0 0 0 0 0 0 0
0 0 - - + + 1 1 1 + + 1 1 1 1 + - 0 0 0 0 0
0 0 - + 1 1 1 1 1 1 1 1 1 1 1 + 0 0 + 0 0 0
0 0 0 0 + 1 1 1 1 1 1 1 1 1 1 1 + + 1 + 0 0
0 0 0 0 + 1 1 + + + 1 1 1 1 1 1 1 1 1 + 0 0
0 0 - + 1 1 + 0 0 0 + 1 1 1 1 1 1 1 + 0 0 0
0 0 0 + 1 1 1 + - 0 0 + 1 1 1 1 1 1 1 + 0 0
0 0 0 0 + 1 1 1 + 0 0 0 + + + + + + + 0 0 0
0 0 0 0 0 + + + - - 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

(d) $E \ominus F \oplus F$

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 # # # # 0 0 0 0 0 0
0 0 0 0 0 0 # 0 0 0 0 # + + + + # 0 0 0 0 0
0 0 0 0 - # + # 0 0 # + 1 1 1 1 + # - 0 0 0
0 0 0 0 # + 1 + # # + 1 1 1 1 + # - - - 0 0
0 0 - 0 # # + 1 + # # + 1 1 + # 0 0 0 0 0 0
0 0 - # + + 1 1 1 + + 1 1 1 1 + # 0 # 0 0 0
0 0 # + 1 1 1 1 1 1 1 1 1 1 1 + # # + # 0 0
0 0 0 # + 1 1 1 1 1 1 1 1 1 1 1 + + 1 + # 0
0 0 0 # + 1 1 + + + 1 1 1 1 1 1 1 1 1 + # 0
0 0 # + 1 1 + # # # + 1 1 1 1 1 1 1 + # 0 0
0 0 # + 1 1 1 + # 0 # + 1 1 1 1 1 1 1 + # 0
0 0 0 # + 1 1 1 + # 0 # + + + + + + + # 0 0
0 0 0 0 # + + + # - 0 0 # # # # # # # 0 0 0
0 0 0 0 0 # # # 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

(e) $E \ominus F \oplus F \oplus F$

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0 1 1 1 1 1 1 0 0 0 0 0
0 0 0 0 0 1 1 1 0 0 1 1 1 1 1 1 0 0 0 0 0 0
0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0
0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0
0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 1 0 0 0
0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 0 0
0 0 0 1 1 1 1 0 0 0 1 1 1 1 1 1 1 1 1 0 0 0
0 0 0 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1 1 1 0 0
0 0 0 0 1 1 1 1 1 0 0 0 1 1 1 1 1 1 1 0 0 0
0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

(f) Final image ($E \ominus F \oplus F \oplus F \ominus F$)

Figure 5. *Boundary line simplification using morphological operators.*

Finally, the *displacement* (or feature transportation) operator will be considered. This is illustrated in Fig.6. Fig. 6(a) shows a line feature to be transported. Fig.6(c) is the result of dilation of map M in Fig.6(a) by structure element G in Fig.6(b). This image is then eroded by another structure element H shown in Fig.6(d) and the result is shown in Fig.6(e), where "R" means that this pixel is to be retained. After cleaning Fig.6(e), the final result is shown in Fig.6(f), i.e. the line feature is displaced one pixel to the upper/right direction.

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

(a) A line to be displaced in image M

```
    1
  1
①
```

(b) Structure element G

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 + + 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 + + + + 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 + + + 1 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 + 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 + 0 + + 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 + + + + 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 1 + + + + + + + + + + 0 0 0 0 0 0
0 0 0 0 0 1 1 + + + + + + + + 0 + + 0 0 0 0
0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 + + 0 + + + 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 + + + 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

(c) Image M dilated by G

```
    1
  ①
1
```

(d) Structure element H

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 + + 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 + + R R 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 + R R R 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 R R 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 R R 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 R 0 + + 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 R R + + 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 1 R R + + + + + + + + 0 0 0 0 0 0
0 0 0 0 0 1 1 R R R R R R R R 0 + + 0 0 0 0
0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 R R 0 + + + 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 R R R 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

(e) The dilated map erosed by H (R=retained)

```
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

(f) The line being one pixel displaced to upper/right

Figure 6. *Line feature displacement using morphological operators.*

Concluding remarks

This paper discussed a few examples, i.e. combination, elimination, simplification and displacement (or transportation), showing the potential applications of morphological tools in generalization of raster map data. No attempt was made to discuss all aspects of such applications. However, the examples do demonstrate the potential power of morphological tools for map generalization. It is especially interesting to see how easy the displacement operation can be accomplished by morphological operator while it is so difficult to do it in vector mode. Further research is being carried out at Curtin University of Technology to develop a series of application models for map generalisation purposes.

As shown, some questions related to structure elements need to be answered before successful application can be made, e.g. what type of structure elements should be used for a particular application and what should be the appropriate size of a structure element. One particular question which is interesting to the author is how to use the natural principle (Li and Openshaw, 1993) as a guide to design structure elements suitable for map generalisation purposes. Research in this area is also being undertaken at the Curtin University of Technology.

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