Morphological Transformation for the Elimination of Area Features in Digital Map Generalization

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For the graphic representation of spatial data (e.g. a map), if the scale of the representation is reduced, then some area features will become too small to be represented, i.e. they need to be eliminated. This elimination procedure is part of the so-called generalization process. This paper describes some techniques in digital map generalization for this procedure, which employ several operators developed in mathematical morphology, a science of shape, form and structure. The techniques include three steps. These involve a process to reduce the size of every area feature using an erosion operator (leading to the disappearance of those small area features which need to be eliminated), a process to recover the size and shape of every area feature that has just been eroded, and a process to simplify the boundaries of recovered area features so as to suit the representation at a smaller scale. The models used in these techniques provide a mathematical basis for area elimination in digital generalization of map and other spatial data. The techniques described in the paper have been tested using examples, which demonstrate the potential for successful application.

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INTRODUCTION

Generalization of digital maps has been regarded by many researchers as a fundamental function in spatial data handling and thus for GIS (e.g. Abler, 1987; Rhind, 1988; Müller et al, 1995). Many research projects on this topic have been carried out world-wide and a number of generalization operations have been identified (e.g. Brassel and Weibel, 1988; Keates, 1989; Shea and McMaster, 1989). However, the current situation is that most of the generalization operations remain at the conceptual level and there is an urgent need to develop those missing algorithms or mathematical models for various generalization operations (Weibel, 1995). This paper describes such missing mathematical models for one of these operations, viz the elimination of small area features.

Elimination of small area features becomes necessary when the physical map space available for detail is reduced as the scale of the spatial data is reduced. Those area features which are too small to be represented need to be eliminated. Figure 1 shows the effect of a scale reduction on small area features. The normal practice for eliminating area features is:

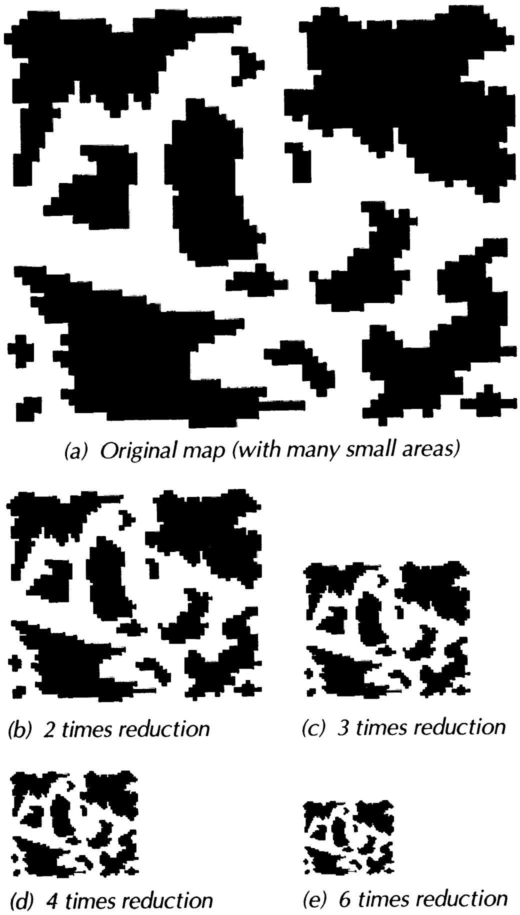
- To compute the size of all area features, and
- To delete all the area features smaller than a given criterion.

Although area elimination seems to be simple, this is not the case in practice, and not much research on this topic has been *done to date.* Only Schylberg (1993) has given a detailed discussion regarding its implementation. He uses rule-based procedures and does not consider the *complexity* of the shapes of those features remaining after the elimination process. However, it is quite often the case that the shape of many area features will appear too irregular after a scale reduction, as can be seen from Figure 1. Therefore, the elimination procedure needs to be followed by a simplification process.

This paper describes some techniques which are able both to eliminate those undesirable small area features and to simplify the boundaries of the remaining area features. The techniques offer a mathematical solution for area elimination, which employs operators (such as erosion and dilation) developed in mathematical morphology, the science of shape, form and structure. They work in raster mode and include three procedures that:

- Reduce the size of every area feature using an erosion operator, leading to the disappearance of those small area features which need to be eliminated,
- Recover the size and shape of every area feature that has just been eroded, and
- Simplify the boundaries of recovered area features so as to suit the representation at a smaller scale.

This introduction is followed by a discussion of the mathematical basis used in this study (mathematical morphology. Then the three steps involved in the elimination process are described, based on the morphological operators.



(a) Original map (with many small areas)

(b) 2 times reduction *(c) 3 times reduction*

(d) 4 times reduction *(e) 6 times reduction*

Figure 1. *Area feature with simple scale reduction.*

MATHEMATICAL MORPHOLOGY AS A TOOLKIT

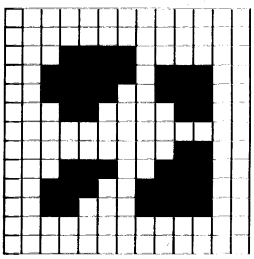
To develop techniques for generalization of spatial data means to build models and procedures for these operations upon the two basic operators developed in mathematical morphology, i.e. dilation and erosion. These can be compared to "+", "-", "x" and "÷" in ordinary algebra. In order to facilitate the discussion of the techniques developed by the authors, the basic concepts in mathematical morphology are introduced here.

Mathematical morphology is a science of shape, form and structure, based on set theory. It was developed by French geostatistical scientists G. Matheron and J. Serra in the 1960s (Matheron, 1975; Serra, 1982). Since then it has found increasing application in digital image processing. Efforts have also been made by researchers to apply morphological tools to map generalization (Li, 1994; Li and Su, 1995; Su and Li, 1995; Su *et al,* 1996) and mapping related sciences such as digital terrain modelling (Li and Chen, 1991). The two basic operators are defined as follows (see Serra, 1982; Haralick *et al,* 1987):

Dilation: $A \oplus B = \{a + b: a \in A, b \in B\} = \cup_{b \in B} A_b$ (1)

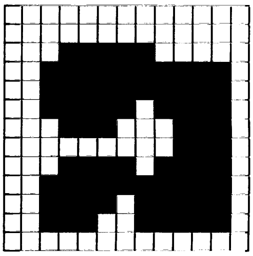
Erosion: $A \ominus B = \{a: a + b \in A, b \in B\} = \cap_{b \in B} A_b$ (2)

where A is the image to be processed and B is called the structuring element, which can be regarded as an analogy to the kernel in convolution operators. In Equation (1), it is called *dilation of A by B* and in Equation (2) *erosion of A by B.* Examples of these two operators are given in Figure 2, where the features are represented by black pixels.

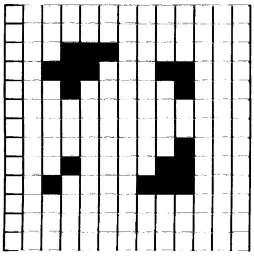


(a) Original image A

(b) The structuring element B



(c) A dilated by B ($A \oplus B$)



(d) A eroded by B ($A \ominus B$)

Figure 2. *Two basic morphological operators: dilation and erosion.*

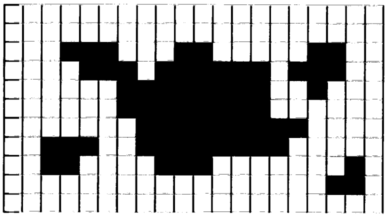
The structuring element is a critical element in any morphological operator. The origin of a structuring element is considered to be its geometric centre if there is no other specific indication. A structuring element can take any shape (square, cross) and size (e.g. 2x2 or 3x3). If a symmetric structuring element is used for dilation, then the shape of the original image will be expanded uniformly along all directions. The dilation in this particular case is called *expansion.* Similarly, the erosion in this case is called *shrink.*

Based on these two basic operators, i.e. dilation and erosion, a number of morphological operators or algorithms have also been developed, such as closing, opening, thinning, thickening, hit or miss, conditional dilation, conditional erosion, conditional thinning, conditional thickening, sequential dilation, conditional sequential dilation, and so on. Among them, the *opening* and *closing* operators are very suitable for the manipulation of area features. These two operators are defined as follows:

Opening: $A \circ B = (A \ominus B) \oplus B$ (3)

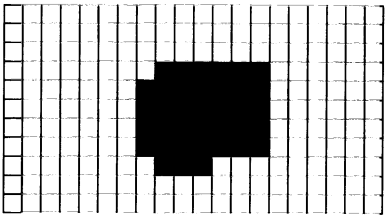
Closing: $A \bullet B = (A \oplus B) \ominus B$ (4)

where A is the original feature and B is the structuring element. Examples of these two operators are given in Figure 3.

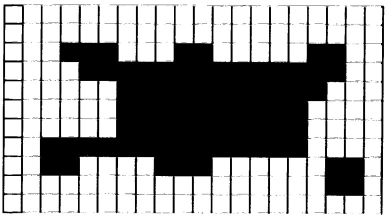


(a) Original feature A

(b) The structuring element B



(c) A opened by B



(d) A closed by B

Figure 3. *Opening and closing operators in mathematical morphology.*

EROSION FOR THE ELIMINATION OF SMALL AREAS

As has been stated previously, the first step involved in the new techniques is to apply an erosion operator to the original map which consists of a number of area features. The model can be written as follows:

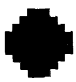
$$E = A \ominus B \qquad (5)$$

where A is the original map, B is the structuring element, and E is the result of the erosion. This process is illustrated in Figure 4 which shows one possible result – some areas being eliminated and others reduced in size. However, the result of an erosion is totally dependent on the size of structuring elements used. Various sized structuring elements are shown in Figure 5.

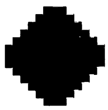
Figure 4. *Result after applying erosion process to area features in Figure 1(a), some eliminated and others reduced in size.*

(a) For 2x reduction

(b) For 3x reduction



(c) For 4x reduction



(d) For 6x reduction

Figure 5. *Structuring elements for various levels of scale reduction.*

As illustrated in Figure 6, the size of the structuring element used in an erosion operator is a critical element and provides direct control over the output of a single erosion operation. Therefore, it is of crucial importance to determine the appropriate size of a structuring element. To do so, scale is a specific factor to be considered.

In other words, the size of the structuring element is dependent on the source scale and the target scale of the spatial data. Its value can be calculated as follows:

$$B_{size} = \frac{S_{target}}{S_{source}} \times D_s \qquad (6)$$

where S_{source} and S_{target} are the scale factors of the source and target data respectively. D_s is the distance at source scale in terms of number of pixels, below which objects on the source map cannot be further represented. This value

(a) Using structuring element in Figure 5(a)

(b) Using structuring element in Figure 5(b)

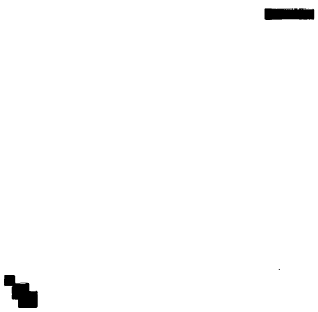
(c) Using structuring element in Figure 5(c)

(d) Using structuring element in Figure 5(d)

Figure 6. *Area features in Figure 1(a) eroded by structuring elements of various size.*

(a) After the first erosion

(b) After the second erosion



(c) After the third erosion

(d) After the fourth erosion

Figure 7. *Area features in Figure 1(a) eroded by successive erosion operators.*

is the *threshold of representation* (equal to or larger than the *threshold of perception*). For example, a forest area is too small to be represented at a map scale of 1:50 000 if the area is smaller than 100 m^2 (LMV, 1985). B_{size} is the size of the structuring element in terms of the number of pixels at target scale.

If a symmetric structuring element with the origin at the centre is to be used, then the dimension of the structuring element should be an odd number. In this case, Equation (6) can be rewritten as follows, where INT means the integer part of the value:

$$B_{size} = INT\left(\frac{INT\left(\frac{S_{target}}{S_{source}} X\, Ds + 0.5\right)}{2}\right) X\, 2 + 1 \quad (7)$$

Of course, the erosion operator can also be repeated several times for a single elimination process. In this case, the number of erosion operations also provides a control over the results of the elimination process. Figure 7 shows this case. In practice, it would be more convenient to apply a single erosion operation to achieve the desired result. Here, the appropriate size of the structuring element to be used is critical.

RECOVERING REDUCED AREA FEATURES

As shown in Figure 6, after an erosion operation, some small area features disappear and large features are reduced in size. Those areas which disappear are areas which one wants to eliminate and those which are reduced in size are what one wants to retain and needs to recover. Two approaches can be used in the recovering process.

The first approach is to apply a restoration process developed by Su and Li (1995). The morphological model is as follows:

$$R_k = (R_{k-1} \oplus B_2) \cap A \quad (8)$$

where A is the original map (before erosion), R_0 = E (eroded result) and B_2 is a squared structuring element with the size 3x3. This is an iteration process. The process will continue until the following condition holds:

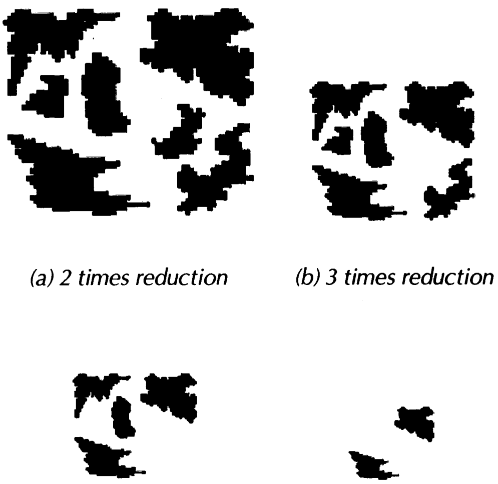
$$R_k = R_{k-1} \quad (9)$$

The results of applying this restoration process to corresponding elements in Figure 6 are shown in Figure 8, with appropriate scale reduction. It can clearly be seen that those area features which are too small to be represented are all eliminated, but those which are large enough to be represented are recovered exactly in terms of both size and shape. However, the shape of these features now appears to be too irregular for the representation at reduced scale. A simplification process will be described in the next section to smooth the boundaries of the area features.

The second method for recovering reduced area features is to apply a dilation process using the same structuring element as for the erosion as follows:

$$R = E \oplus B_2 \quad (10)$$

where E is the map obtained after an erosion by B (Equation 5), and B_2 is a structuring element. When B = B_2, Equation (5) and Equation (10) together become an opening operator. The results of applying this process to Figure 6 are shown in Figure 9.



(a) 2 times reduction *(b) 3 times reduction*

(c) 4 times reduction *(d) 6 times reduction*

Figure 8. *Elimination of area features for scale reduction at different levels, with features then recovered by a restoration process.*

It seems that the boundaries of the area features in Figure 9 are much smoother than those shown in Figure 8. Indeed, some of them are very smoothed already. However, a close examination reveals that some parts of their boundaries are also quite irregular and a smoothing process might be desirable.

(a) 2 times reduction *(b) 3 times reduction*

(c) 4 times reduction *(d) 6 times reduction*

Figure 9. *Elimination of area features for scale reduction at different levels, with features then recovered by a dilation process.*

SIMPLIFICATION PROCESS FOR SMOOTHING AREA BOUNDARIES

As indicated in the previous section, the boundaries of the area features will appear too irregular for representation at a smaller scale. This is especially so in Figure 8. To overcome this deficiency, an opening operator and then a closing operator can be applied to the results obtained from the recovering process.

$$F = (R \circ B_3) \bullet B_3 \qquad (11)$$

where B_3 is a 3x3 squared structuring element, R represents the recovered area features, and F is the final result. The result F of applying Equation (11) to Figure 8 is shown in Figure 10.

In the case of areas recovered by a dilation process, a closing operator can be applied.

$$F = R \bullet B_3 = (R \oplus B_3) \ominus B_3 \qquad (12)$$

The result of applying Equation(12) to Figure 9 is shown in Figure 11.

(a) 2 times reduction *(b) 3 times reduction*

(c) 4 times reduction *(d) 6 times reduction*

Figure 10. *Final results for area elimination for the restoration approach.*

(a) 2 times reduction *(b) 3 times reduction*

(c) 4 times reduction *(d) 6 times reduction*

Figure 11. *Final results for area elimination for the opening approach.*

CONCLUSIONS

In this paper, new techniques consisting of three steps, erosion, recovering and simplification, have been described for the elimination of area features. They are built upon the operators developed in mathematical morphology. It is clear that, after this three step process, when compared to simple scale reduction only, the features are clear and simplified, thus suitable for representation at smaller scales. In fact, the results are very promising.

The techniques described in this paper seem quite simple. The key to success is the determination of the shape and size of the structuring elements used. The relationship between the size of structuring elements and the scales of spatial data also need to be considered. In addition, the shape of structuring elements needs to be adapted to the appearance of the area features. Circular structuring elements should be used for curved area features and rectangular structuring elements for rectangular area features.

Finally, it should be noted that this paper deals only with geometric issues. Indeed, the main aim of this paper is to offer a mathematically elegant solution for the geometric transformations involved in generalization. However, if these transformation models are to be employed in a generalization system, then semantic information should also be considered as the control over them.

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REFERENCES

Abler, R. (1987) The National Science Foundation National Center for Geographic Information and Analysis, *International Journal of Geographic Information Systems,* Vol. 1, No. 4, pp. 303–326.

Brassel, K. and Weibel, R. (1988) A review and conceptual framework of automated map generalization, *International Journal of Geographical Information Systems,* Vol. 1, No. 4, pp. 303–326.

Haralick, R., Sternberg, S. and Zhuang, X. (1987) Image analysis using mathematical morphology, *IEEE Transactions of Pattern Analysis and Machine Intelligence,* Vol. 9, No. 4, pp. 532–550.

Keates, J. S. (1989) *Cartographic Design and Production,* 2nd Edition, John Wiley and Sons, New York, 261 pp.

Li, D. and Chen, X-Y. (1991) Automated generating triangulated irregular digital terrain model networks by mathematical morphology, *ISPRS Journal of Photogrammetry and Remote Sensing,* No. 46, pp. 283–295.

Li, Z. (1994) Mathematical morphology in digital generalization of raster map data, *Cartography,* Vol. 23, No. 1, pp. 1–10.

Li, Z. and Su, B. (1995) Algebraic models for feature displacement in the generalization of digital map data using morphological techniques, *Cartographica,* Vol. 32, No. 3, pp. 39–56.

LMV (1985) *Handbook–Topographic map,* National Land Survey of Sweden, Gävle, Sweden (in Swedish).

Matheron, G. (1975) *Random Sets and Integral Geometry,* John Wiley and Sons, New York, 261 pp.

Müller, J.-C., Weibel, R., Lagrange, J. P and Salge, F. (1995) Generalization: state of the art and issues, In: *GIS and Generalization,* Müller J.-C., Lagrange J. P. and Weibel R. (eds), Taylor and Francis, London, pp. 3–17.

Rhind, D. (1988) A GIS research agenda, *International Journal of Geographical Information Systems,* Vol. 2, No. 1, pp. 23–28.

Schylberg, L. (1993) *Computational Methods for Generalization of Cartographic Data in a Raster Environment,* Doctoral Thesis, Royal Institute of Technology, Sweden, 137 pp.

Serra, J. (1982) *Image Processing and Mathematical Morphology,* Academic Press, New York, 610 pp.

Shea, K. and McMaster, R. (1989) Cartographic generalization in a digital environment: when and how to generalize, *Proceedings of Auto-Carto 9,* Baltimore, Maryland, March, pp. 57–67.

Su, B. and Li, Z. (1995) An algebraic basis for digital generalization of area-patches based on morphological techniques, *The Cartographic Journal,* Vol. 32, No. 2, pp. 148–153.

Su, B., Li, Z., Lodwick, G. and Müller, J.-C. (1997) Algebraic models for the aggregation of area features based upon morphological operators, *International Journal of Geographical Information Systems,* Vol. 11, No. 3, pp. 233–246.

Weibel, R. (1995) *Summary report: workshop on progress in automated map generalization,* Technical Report, ICA Working Group on Automated Map Generalization, Barcelona, Spain, September, 3pp.