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A Voronoi-based spatial algebra for spatial relations*

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Abstract Spatial relation between spatial objects is a very important topic for spatial reasoning, query and analysis in geographical information systems (GIS). The most popular models in current use have fundamental deficiencies in theory. In this paper, a generic algebra for spatial relations is presented, in which (i) appropriate operators from set operators (i.e. union, intersection, difference, difference by, symmetric difference, etc.) are utilized to distinguish the spatial relations between neighboring spatial objects; (ii) three types of values are used for the computational results of set operations—content, dimension and number of connected components; and (iii) a spatial object is treated as a whole but the Voronoi region of an object is employed to enhance its interaction with its neighbours. This algebra overcomes the shortcomings of the existing models and it can effectively describe the relations of spatial objects.

Keywords: **spatial relations, Voronoi-based algebra, spatial algebra, topological relations.**

A formal theory of description and determination for relations between spatial objects is usually important to spatial query, analysis and reasoning in the GIS field. For example, in the case of digital map generalization, spatial relations between map objects will be altered after applying generalization operations such as selective omission, aggregation, displacement, exaggeration and so on. These relations could be metric and/or topological. Due to such changes, spatial conflicts may be created by these operations and such conflicts need to be checked and resolved. Recently, much attention has been paid to this topic[1]. For such a purpose, appropriate mathematical models for spatial relations are required.

The importance of spatial relations theory was recognized in the early 1980s[2,3]. Since then, many papers on this topic have been published by researchers from the computing science and GIS communities. The approaches used in these works can be classified into two categories, i.e. decomposition-based and whole-based[4]. In the former, a spatial object is represented in terms of the set of its components, and relations are described and determined by the combinatorial relations of those components. In the latter, the spatial object is considered as a whole, and spatial relations between spatial objects are described and determined by the interaction between whole bodies of these objects instead of their components.

In the category of decomposition-based approaches, most models are built upon point set topology[5~8]. In these models, two or three components of a spatial object, i.e. interior, boundary and exterior, are utilized. The most fundamental one is the 4-intersection model[9], making use of the interior and boundary of a spatial object. Later, this model was extended to a 9-intersection model[10] in which the exterior of a spatial object is also included. These models have been implemented in raster mode[11,12], through the use of vector representation for raster cells. However, there is a fundamental deficiency in theory for either 4- or 9-intersection models. The 9-intersection model was later modified by Chen and his collaborators[13], with the exterior of a spatial object replaced by its Voronoi region. However, the modified model is still not generic enough.

In the category of the whole-based approach, the main work includes the spatial logic model developed by Randell et al.[14] and temporal logic model by Allen[15]. The former is built upon the calculus of individuals[16] which is in turn based on "region connection" ontology. In this model, a set of topological relationships between concave regions is axiomatized. Some other work based on Clarke's region theory can

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also be found in the literature[17]. The main advantage of such work rests in its rigorous logic so as to facilitate the mathematical deduction and proof. Such models have mainly been applied to objects of regions in the context of artificial intelligence. Allen's temporal logic relations model[3] was originally put forward to handle two-dimensional temporal space but it was later widely extended to higher dimensional space[18~21]. Unfortunately, most of these models are limited to the point-based abstraction of spatial objects.

In spite of these efforts and progress made in the last decade, it is undeniable that there are many imperfections associated with existing work, as will be discussed in Section 2. In other words, the formal description and determination of spatial relations are still an open issue deserving further research, as pointed out by Vasilis et al.[22]. In this paper, an alternative approach is proposed, i.e. an algebraic approach based on Voronoi regions. This is a whole-based approach. In order to capture the interaction among whole objects, the Voronoi regions of spatial objects are utilized.

1 The strategies used in this study

In order to develop an appropriate strategy for this study, a critical examination of existing models seems appropriate.

1.1 Fundamental deficiency in theory associated with existing models

As mentioned in the previous section, in the decomposition-based approach, a spatial object is decomposed into several topological components such as the interior and boundary. The interaction between these object components determines the relations between spatial objects. Typical examples are the 4-intersection model[9] and its extension to the 9-intersection model[10]. However, there are many theoretical problems associated with this model[13,5,23].

The fundamental deficiency in theory with the 4-intersection model (Eq. (1)) is the inconsistency in the definition of a line object. In 1-D space, the two end points define the boundary of the line. However, this definition is not valid any more in a 2-D space. If one simply adopts this definition from a 1-D to 2-D space, a topological paradox will be caused, i.e. interior meeting exterior, as shown in Fig. 1. This problem was closely examined by Li et al.[24] and no further explanation will be given.

$$R_4(A,B) = \begin{pmatrix} A^0 \cap B^0 & A^0 \cap \partial B \\ \partial A \cap B^0 & \partial A \cap \partial B \end{pmatrix}. \quad (1)$$

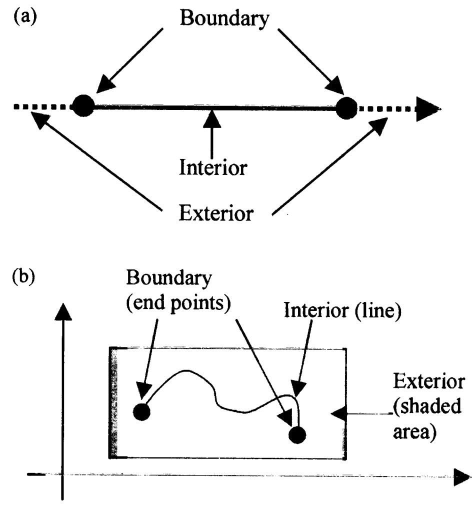


Fig. 1. Fundamental deficiency in theory associated with the 4-intersection model, i.e. a topological paradox being caused. (a) Topological components of a line in 1-D vector space; (b) topological paradox is caused if the boundary definition of a line in 1-D is simply adopted in 2-D space, i.e. interior meeting exterior.

The fundamental deficiency in theory with the 9-intersection model (Eq. (2)) is the linear dependency between the three topological components, i.e. interior, boundary and exterior (complement). That is, for an object A, the interior A^0, boundary ∂A and the complements A^- together form the complete study space C. Mathematically, $C = A^0 + \partial A + A^-$ or $A^- = C - (A^0 + \partial A)$. The model given by Eq. (2) could then be written as Eq. (3). As the size of a given study area is fixed, C is a constant. As a consequence, the complement of an object is linearly dependent on its interior and boundary. Therefore, the extension from the 4- to the 9-intersection model is invalid.

$$R_9(A,B) = \begin{pmatrix} A^0 \cap B^0 & A^0 \cap \partial B & A^0 \cap B^- \\ \partial A \cap B^0 & \partial A \cap \partial B & \partial A \cap B^- \\ A^- \cap B^0 & A^- \cap \partial B & A^- \cap B^- \end{pmatrix}. \quad (2)$$

$$R_9(A,B) = \begin{pmatrix} A^0 \cap B^0 & A^0 \cap \partial B & A^0 \cap (C - B^0 - \partial B) \\ \partial A \cap B^0 & \partial A \cap \partial B & \partial A \cap (C - B^0 - \partial B) \\ (C - A^0 - \partial A) \cap B^0 & (C - A^0 - \partial A) \cap \partial B & (C - A^0 - \partial A) \cap (C - B^0 - \partial B) \end{pmatrix}. \quad (3)$$

To solve the linear dependency problem, Chen et al.[13] have used the Voronoi regions of A and B to replace their complement. However, the fundamental problem associated with the definition of a line is still unsolved.

Now let us turn to the whole-based approaches. The whole-based approach directly uses spatial objects instead of their components for the description of spatial relations, thus the problems associated with the decomposition-based approach can be avoided. However, it is not sufficient to consider only objects themselves for distinguishing spatial relations[25]. Indeed, many relations cannot be distinguished if only the objects themselves are used.

1.2 Strategies used in this study

For the reasons mentioned above, in this study, the whole-based approach will be used as a basis. In order to overcome the shortcomings of the whole-based approach, an additional parameter should be introduced and this parameter must meet the following criteria: (i) being insensitive to the dimensionality of space; and (ii) being closely related to the object so as to have functions similar to those of boundary and exterior.

As a result, the Voronoi region (Fig. 2) of an object is selected, for it has many good properties[26,27]. A Voronoi region or Thiessen polygon for a point is the locus of points closer to that point than to any other given one[28].

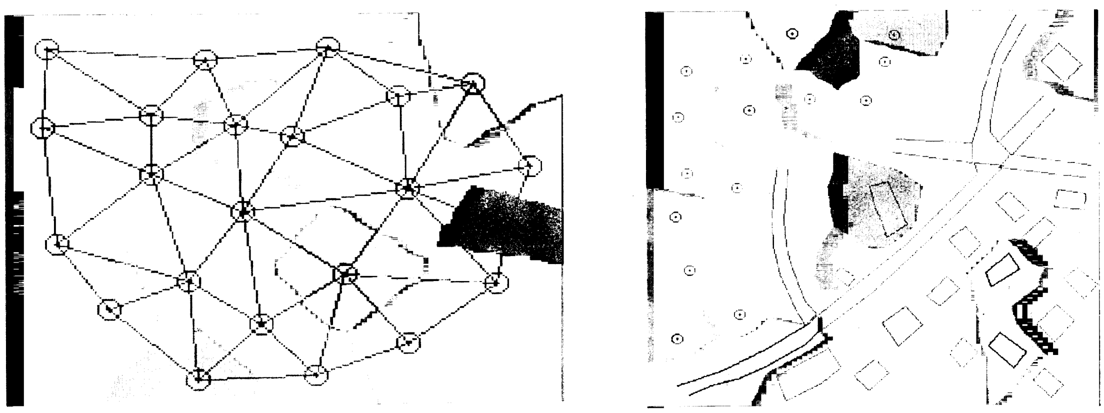


Fig. 2. The Voronoi regions of a set of spatial objects and associated tessellation. Left: Voronoi regions of point sets; right: Voronoi regions of spatial objects.

Another observation arising from the analysis of existing literature is that, out of the many set operators, only the "intersection" operator has been utilized except for the work by Galton[29]. This is perhaps the most expensive one in terms of computation. There is no reason why other operators cannot be used. Therefore, it is attempted to explore the full range of set operators to constitute a spatial algebra for the spatial relations.

In summary, the basic strategies adopted here are: (i) a spatial object is treated as a whole; (ii) the Voronoi region of an object is employed to enhance its interconnection with neighbors; (iii) the appropriate operators from set operators are utilized to distinguish the spatial relations between neighboring spatial objects; and (iv) several types of values are used for the computational results of set operations, e.g. content, dimension and number of connected components and so on.

2 A spatial algebra for spatial relations: A generalization

In this section, a number of set operators will be employed and spatial concepts are embedded into the algebra.

2.1 A spatial algebra for spatial relations based on set operators

Spatial objects are often regarded as sets in space in the context of GIS. This is very important as it means that objects can be manipulated by ordinary set operators: union, intersection, set and symmetrical differences, and so on. At the same time, spatial relations can be considered as the result of handling these "sets". In fact, the theory of sets is the basis of the description and determination of spatial relations; in particular, topological relations can be regarded as detailed relations between sets[30]. Fig. 3 illustrates working principles of set operators, using line and area objects as examples.

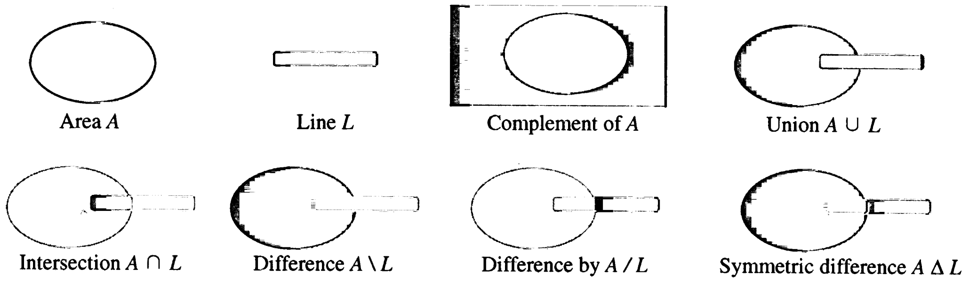


Fig. 3. Set operators for spatial objects.

From the viewpoint of algebra, these set operators together with spatial concepts (see sub-section 2.2) form an algebra, called the spatial algebra, for spatial relations in this study. Let O denote the set of all spatial objects, then the spatial relation, $B(a, b)$, between object a and object b in set O can be represented by

$$B(a,b) = f(a\theta b) = f(a \cup b, a \cap b, a \setminus b, a/b, a\Delta b \cdots) \quad (4)$$

where θ denotes the above set of set operators, i.e. $\theta = \{\cup, \cap, \setminus, /, \Delta\cdots\}$, representing union, intersection, difference, difference by, symmetric difference, etc. If the desired relation can be sufficiently described by one operator, then other operators may be omitted. f is a function to take a type of value for the results of set operations (see Eq. (4)).

Eq. (3) can be regarded as a simple spatial algebra for the description of spatial relations. Spatial relations can be distinguished on a coarse level with this equation. For example, topological relations varying from disjoint relations to equal relations between two solid area objects can be determined by this equation, see Section 4. However, this equation is not able to describe some more detailed spatial relations. This is because spatial relations are not only dependent on objects themselves but also on their surrounding space as stated in the previous sections.

Currently, only the intersection operator is widely used for the determination and description of spatial relations, mainly topological relations and order relations. In fact, some relations may be easily distinguished by other operators but not by the intersection operator. Fig. 4 shows such an example which illustrates the superiority of "difference" operator over intersection. It is clear that "overlap" and "contained by" can be easily distinguished by their "difference" but not their intersection.

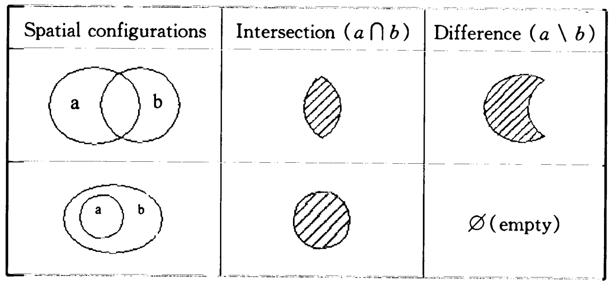


Fig. 4. Results from "difference" and "intersection" operators. (The former is able to distinguish "overlap" and "contain", but the latter is not).

2.2 Three types of values for the results of set operators in the spatial algebra

The value of $(a\theta b)$ can take three different forms, i.e. content, dimension, and the number of connected components.

Fig. 5 shows these values in the case of intersection operation. "Content" is a quality measure, i.e. either "empty" or "non-empty". "Dimension" is a quantitative measure, i.e. either 0-dimensional (point), 1-dimensional (line) or 2-dimensional. For the case of "empty", a dimensional number of (-1) is usually used. "Number of connected components" is quantitative measure at a finer level. In the case of "empty", the number is 0. Otherwise, the number could be any integer larger than 0. For example, two

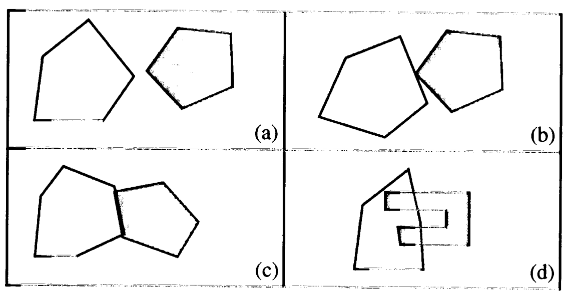


Fig. 5. Content, dimension and the number of components of intersection operator. (a) empty intersection; (b) 0-dimensional intersection with one connected component; (c) 1-dimensional intersection with one connected component; (d) 2-dimensional intersection with two connected components.

objects "a" and "b" have 2-dimensional overlap with two parts connected, as shown in Fig. 5(d).

Mathematically, the value of each element in the $(a\theta b)$ set, say e, could be denoted by

$$e \in \begin{cases} \{\varnothing, -\varnothing\} \text{ if } f \text{ is a function to take content, donated by } f_C; \\ \{-1, 0, 1, 2, \cdots\} \text{ if } f \text{ is a function to take dimension, donated by } f_D; \\ \{0, 1, 2, 3, \cdots\} \text{ if } f \text{ is a function to take connected number, donated by } f_N. \end{cases} \quad (5)$$

As a result, a spatial relation shown in Fig. 5(a) could be represented as $B(a, b) = f_C(a\theta b) = (-\varnothing, \varnothing, -\varnothing, -\varnothing, -\varnothing)$, if $B(a, b)$ takes content as the type of value for the spatial algebra. If the result of operators consists of multiple parts, then the highest dimension should be used for the value of the $f_D(a\theta b)$ function. In addition, a combination of dimension and connected number values could also be used to form a value set, (f_D, f_N). For example, such a set for Fig. 5(d) could be represented as $B(a, b) = ((2,1), (2,2), (2,1), (2,1), (2,1))$.

As the content, dimension and number represent three different levels from coarse to fine, it is quite possible that content is enough to represent a particular spatial relation. In such a case, it is unnecessary to consider dimension or connected numbers. On the other hand, it is also possible that a spatial relation cannot be sufficiently described even if dimension is used. In this case, connected number should also be considered.

3 Voronoi-based spatial algebra for spatial relations: Further extension

In the previous section, a simple spatial algebra is developed for spatial relations. However, as will be discussed later, some spatial relations will be confused if only the spatial objects are used. In order to make the spatial algebra more general, Voronoi regions of spatial objects are introduced into the model expressed in Eq. (1).

3.1 Voronoi region as a topological component of a spatial object

Spatial relations essentially reflect the spatial configuration between objects. In other words, for individual object, the surrounding space must also be taken into account in addition to the surrounding objects if sound models for spatial relations are to be developed. The role of Voronoi region in this study serves the purpose of tightening the inter-relation among a spatial object and its neighbouring objects and space.

A Voronoi region describes the spatial proximity or influent region of a spatial object. The Voronoi regions of all spatial objects together will form a tessellation of space. This tessellation is called Voronoi diagram. There are also other names but such discussion and other topics could be found elsewhere[27]. The dual graph is the well-known Delaunnay triangulation network in GIS and computational geometry. Fig. 2 illustrates Voronoi regions, Voronoi diagram and the corresponding Delaunnay triangulation of a point set. Fig. 6 shows Voronoi regions of two objects with two different kinds of spatial configurations.

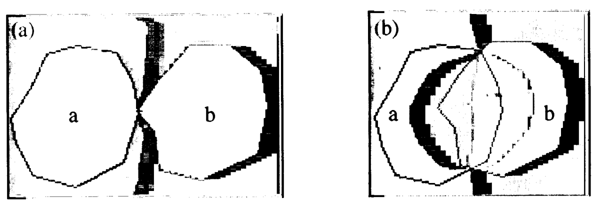


Fig. 6. Voronoi regions of spatial objects with complex configurations. (a) Touch (meet); (b) overlap.

It is clear that the Voronoi region of a spatial object could serve for two purposes, i.e. to connect spatial objects together to form a space tessellation, and to serve as a confined exterior of the spatial object at the same time. Therefore, Voronoi region is introduced into the spatial algebra for spatial relations.

3.2 Voronoi-based spatial algebra: further extension

Let a^V be the Voronoi region of spatial object a and b^V be the Voronoi region of spatial object b, then the spatial relation $B(a, b)$ between object a and object b can be listed in Table 1 concisely, which can be expanded into a matrix form as in Table 2.

Table 1. The concise representation of the new algebraic model

$B(a,b) = F(A^T\theta B)$	b	b^V
a	$(a\theta b)$	$(a^V\theta b^V)$
a^V	$(a^V\theta b)$	$(a^V\theta b^V)$

Table 2. The extended form of the algebraic model based on Voronoi regions

$B(a,b)=F(A^{T}\theta B)$	(a,b)	(a,b^{V})	(a^{V},b)	(a^{V},b^{V})
Union $\cup$	$a\cup b$	$a\cup b^{V}$	$a^{V}\cup b^{V}$	$a^{V}\cup b^{V}$
Intersect $\cap$	$a\cap b$	$a\cap b^{V}$	$a^{V}\cap b^{V}$	$a^{V}\cap b^{V}$
Difference $\setminus$	$a\setminus b$	$a\setminus b^{V}$	$a^{V}\setminus b^{V}$	$a^{V}\setminus b^{V}$
Difference by /	a/b	a/b^{V}	a^{V}/b^{V}	a^{V}/b^{V}
Symmetric difference Δ	$a\Delta b$	$a\Delta b^{V}$	$a^{V}\Delta b^{V}$	$a^{V}\Delta b^{V}$

Mathematically, let $A=[a,a^{V}]$ and $B=[b,b^{V}]$, then the relations could be described by the following equation:

$$B(a,b)=F(A^{T}\theta B)=F\{[a,a^{V}]^{T}\theta[b,b^{V}]\}$$
$$=F\begin{pmatrix}(a\theta b) & (a\theta b^{V})\\ (a^{V}\theta b) & (a^{V}\theta b^{V})\end{pmatrix} \tag{6}$$

where, F is a function similar to the f in Eq. (4). Generally speaking, the following function is sufficient:

$$B'(a,b)=F\{(a\theta b),(a^{V}\theta b^{V})\} \tag{7}$$

In practice, if a spatial relation can be sufficiently described by $(a\theta b)$, the other operand, i. e. $(a^{V}\theta b^{V})$, may be ignored. As a result, spatial relations can be described in a flexible manner.

4 Topological relations with the spatial algebra

4.1 Assumptions used in the algebra

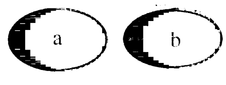
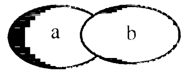
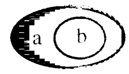
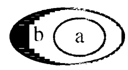
In fact, not all the values of Eq. (6) and/or Table 2 are valid in practical applications. A number of assumptions can be made for the determination of the useful values in Eq. (6), and Tables 1 and 2. These assumptions are formulated by considering a number of factors, i. e. the properties of spatial objects, the embedding space, the relations between selected operators in the model and so on. These assumptions are listed as follows: (i) spatial objects are embedded in Euclidean space; and (ii) a spatial object has only one connected component.

4.2 Topological relations between simple area and line objects

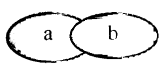
The topological relations between simple area objects are illustrated in Table 3. The topological relations between simple lines are illustrated in Table 4 in the matrix form with multiple operators. Here, in order to distinguish the "meet" and "intersect" relations between lines, the value of the connected components of $(a^{V}\theta b^{V})$ is used. In fact, different types of "meet" relations can also be distinguished by this operation. It is noted that the kinds of relations listed in Table 4 are very difficult for other models to distinguish.

Table 3. Topological relations between solid area objects

(a) Basic relations between solid area by empty/non-empty

f_C	$a\cup b$	$a\cap b$	$a\setminus b$	a/b	$a\Delta b$	Semantic
	$-\varnothing$	$-\varnothing$	$-\varnothing$	$-\varnothing$	$-\varnothing$	Disjoint
	$-\varnothing$	$-\varnothing$	$-\varnothing$	$-\varnothing$	$-\varnothing$	Meet/Overlap
	$-\varnothing$	$-\varnothing$	$-\varnothing$	$-\varnothing$	$-\varnothing$	Contain/Cover
	$-\varnothing$	$-\varnothing$	$-\varnothing$	$-\varnothing$	$-\varnothing$	Contained/Covered
	$-\varnothing$	$-\varnothing$	$-\varnothing$	$-\varnothing$	$-\varnothing$	Equal

(b) Further discrimination of meet/overlap with the simple model by dimensions

f_D	$a\cup b$	$a\cap b$	$a\setminus b$	a/b	$a\Delta b$	Semantic
	2	0	2	2	2	0-D meet
	2	1	2	2	2	1-D meet
	2	2	2	2	2	Overlap

(c) Further discrimination of contain/cover and contained/covered with the general model

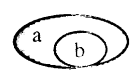
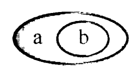
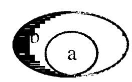
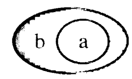
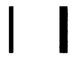
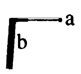
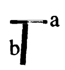
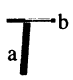
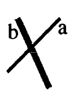
F_C	$a\cup b$ / $a^{V}\cup b^{V}$	$a\cap b$ / $a^{V}\cap b^{V}$	$a\setminus b$ / $a^{V}\setminus b^{V}$	a/b / a^{V}/b^{V}	$a\Delta b$ / $a^{V}\Delta b^{V}$	Semantic
	$-\varnothing$	$-\varnothing$	$-\varnothing$	$\varnothing$	$-\varnothing$	Cover
	$-\varnothing$	$-\varnothing$	$-\varnothing$	$\varnothing$	$-\varnothing$	
	$-\varnothing$	$-\varnothing$	$-\varnothing$	$\varnothing$	$-\varnothing$	Contain
	$-\varnothing$	$\varnothing$	$-\varnothing$	$\varnothing$	$-\varnothing$	
	$-\varnothing$	$-\varnothing$	$\varnothing$	$-\varnothing$	$-\varnothing$	Covered
	$-\varnothing$	$-\varnothing$	$\varnothing$	$-\varnothing$	$-\varnothing$	
	$-\varnothing$	$-\varnothing$	$\varnothing$	$-\varnothing$	$-\varnothing$	Contained
	$-\varnothing$	$\varnothing$	$\varnothing$	$-\varnothing$	$-\varnothing$	

Table 4. Topological relations between simple lines

(a) Level 1: basic relations with the simple model by empty/non-empty

f_D	$a \cup b$	$a \cap b$	$a \setminus b$	a/b	$a \Delta b$	Semantic
	>-1	-1	>-1	>-1	>-1	Disjoint
	>-1	0	>-1	>-1	>-1	Meet/Intersect
	>-1	1	>0	>0	>0	Overlap
	>-1	>-1	>-1	-1	>-1	Contain
	>-1	>-1	-1	>-1	>-1	Contained
	>-1	>-1	-1	-1	-1	Equal

(b) Level 2: the description of meet/intersect with the general model

F_N	$a \cup b$ / $a^V \cup b^V$	$a \cap b$ / $a^V \cap b^V$	$a \setminus b$ / $a^V \setminus b^V$	a/b / a^V/b^V	$a \Delta b$ / $a^V \Delta b^V$	Semantic
	1	1	1	1	2	Meet at ends
	1	2	1	1	2	
	1	1	2	1	3	Meet at b's end and a's middle
	1	2	1	1	2	
	1	1	1	2	3	Meet at a's end and b's middle
	1	2	1	1	2	
	1	1	2	2	4	Meet at middles
	1	2	1	1	2	
	1	1	2	2	4	Intersect
	1	4	2	2	4	

4.3 Topological relations between simple objects with different dimensions

Topological relations between simple objects with different dimensions here mainly refer to those between areas and lines, areas and points and lines with points. In fact, the distinction of relations between lines and areas can be realized without the aid of such notions as interior, exterior and boundary. This is an important characteristic of this model, which makes its realization easy and practical in either raster space or rector space. These relations are illustrated in Tables 5~7.

Table 5. Topological relations between areas and lines

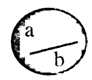
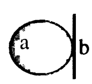
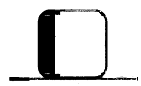
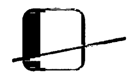
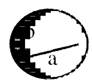
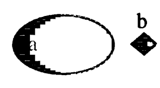
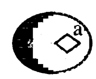
F_D	$a \cup b$ / $a^V \cup b^V$	$a \cap b$ / $a^V \cap b^V$	$a \setminus b$ / $a^V \setminus b^V$	a/b / a^V/b^V	$a \Delta b$ / $a^V \Delta b^V$	Semantic
	>-1	-1	>-1	>-1	>-1	Disjoint
	2	$>=-1$	2	2	2	
	>-1	>-1	>-1	-1	>-1	Contain
	2	-1	2	-1	2	
	>-1	0	>-1	>-1	>-1	0-D meet
	2	1	2	2	2	
	>-1	1	>-1	>-1	>-1	1-D meet
	2	2	2	2	2	
	>-1	1	>-1	>-1	>-1	Intersect
	2	1	2	2	2	
	>-1	>-1	>-1	>-1	>-1	Contained
	2	-1	-1	2	2	

Table 6. Topological relations between points and lines

f_C	$a \cup b$	$a \cap b$	$a \setminus b$	a/b	$a \Delta b$	Semantic
	$-\varnothing$	$\varnothing$	$-\varnothing$	$-\varnothing$	$-\varnothing$	Disjoint
	$-\varnothing$	$-\varnothing$	$-\varnothing$	$\varnothing$	$-\varnothing$	Contain
	$-\varnothing$	$-\varnothing$	$\varnothing$	$-\varnothing$	$-\varnothing$	Contained

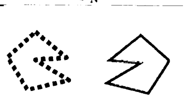
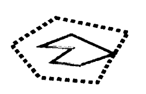
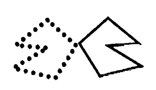
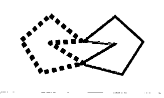
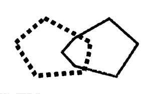
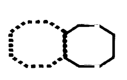
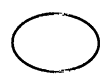
Table 7. Topological relations between points and areas

f_C	$a \cup b$	$a \cap b$	$a \setminus b$	a/b	$a \Delta b$	Semantic
	$-\varnothing$	$\varnothing$	$-\varnothing$	$-\varnothing$	$-\varnothing$	Disjoint
	$-\varnothing$	$-\varnothing$	$-\varnothing$	$\varnothing$	$-\varnothing$	Contain
	$-\varnothing$	$-\varnothing$	$\varnothing$	$-\varnothing$	$-\varnothing$	Contained

4.4 Topological relations between complex objects

The description of spatial relations among loop line objects has been a difficult task and there is a lack of efficient solutions. But this kind of relation may also be distinguished by the new approach. In order to describe this kind of relation, the value of combination of dimension and connected number of $(a\theta b)$ as well as the value of connected number of $(a^V \theta b^V)$ is employed. The result is shown in Table 8.

Table 8. Spatial relations between loop line objects

(F_D, F_N)	$a \cup b$	$a \cap b$	$a \setminus b$	a / b	$a \Delta b$	Semantic
F_N	$a^V \cup b^V$	$a^V \cap b^V$	$a^V \setminus b^V$	a^V / b^V	$a^V \Delta b^V$	
	(1, 2)	(−1, −1)	(1, 1)	(1, 1)	(1, 2)	Disjoint
	3	1	2	2	4	
	(1, 1)	(0, 1)	(1, 1)	(1, 1)	(1, 2)	Interior meet
	3	1	2	2	4	
	(1, 1)	(0, 1)	(1, 1)	(1, 1)	(1, 2)	Exterior meet
	3	2	2	2	4	
	(1, 1)	(0, 2)	(1, 2)	(1, 2)	(1, 4)	Exterior meet
	5	3	3	3	6	
	(1, 1)	(0, 2)	(1, 2)	(1, 2)	(1, 4)	Intersect
	5	5	4	4	8	
	(1, 1)	(1, 1)	(1, 1)	(1, 1)	(1, 2)	Overlap
	3	2	2	2	4	
	(> −1, 1)	(> −1, 1)	(−1, 1)	(−1, 1)	(−1, 1)	Equal
	2	2	−1	−1	−1	

Using the new approach without other extension, the distinction of complex relations between area objects can also be realized, including various "inside" relations.

5 Conclusions

In this paper, a novel approach for the description of spatial relations is employed. It consists of three strategies as follows: (i) appropriate operators from set operators (i. e. union, intersection, difference, difference by, symmetric difference, etc.) are utilized to distinguish the spatial relations between neighbouring spatial objects; (ii) three types of values are used for the computational results of set operations— content, dimension and number of connected components; (iii) a spatial object is treated as a whole but the Voronoi region of an object is employed to enhance its interaction with its neighbours.

This approach combines the advantages of both the decomposition-based and whole-based approaches. With this strategy, a generic algebraic model is developed to distinguish and determine spatial relations between objects in geographical databases. Such a model includes mainly three integrands, i. e. spatial objects themselves, their Voronoi regions, and proper set operators. Spatial objects here mainly refer to points, lines and areas in planar space. They can be considered as basic spatial data types and fundamental abstractions in modelling spatial databases. The set operators are primitive operations in GIS, especially in raster-based systems. From a theoretical point of view, this model is a more general model than existing models based on both the whole-based and decomposition-based approaches. It also overcomes the fundamental deficiencies in theory associated with existing models. With this model, spatial relations can be described hierarchically from coarse to detailed level with the aid of Voronoi regions as well as the three types of values. This model is also able to discriminate the "disjoint" relation with a higher resolution. From a practical viewpoint, using this approach, the integration of the description and computation of spatial relations in both vector and raster space is realized in a natural way.

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