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Research Article

A Voronoi-based 9-intersection model for spatial relations

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Abstract. Models of spatial relations are a key component of geographical information science (GIS). Efforts have been made to formally define spatial relations. The foundation model for such a formal presentation is the 4-intersection model proposed by Egenhofer and Franzosa (1991). In this model, the topological relations between two simple spatial entities A and B are transformed into point-set topology problems in terms of the intersections of A's interior and boundary with B's interior and boundary. Later, Egenhofer and Herring (1991) extended this model to 9-intersection by addition of another element, i.e. the exterior of an entity, which is then defined as its complement. However, the use of its complement as the exterior of an entity causes the linear dependency between its interior, boundary and exterior. Thus such an extension from 4- to 9-intersection should be of no help in terms of the number of relations. This can be confirmed by the discovery of Egenhofer *et al.* (1993). The distinction of additional relations in the case where the co-dimension is not zero is purely due to the adoption of definitions of the interior, boundary and exterior of entities in a lower dimensional to a higher dimension of space, e.g. lines in 1-dimensional space to 2-dimensional space. With such adoption, the topological convention that the boundary of a spatial entity separates its interior from its exterior is violated. It is such a change of conventional topological properties that causes the linear dependency between these three elements of a spatial entity (i.e. the interior, boundary and exterior) to disappear, thus making the distinction of additional relations possible in such a case (i.e. the co-dimension is not zero).

It has been discussed that the use of Voronoi-regions of an entity to replace its complement as its exterior in the 9-intersection model would solve the problem (i.e. violation of topological convention) or would make this model become more comprehensive. Therefore, a Voronoi-based 9-intersection model is proposed. In addition to the improvement in the theoretical aspect, the Voronoi-based 9-intersection model (V9I) can also distinguish additional relations which are beyond topological relations, such as high-resolution disjoint relations and relations of complex spatial entities. However, high-resolution disjoint relations defined by this model are not purely topological. In fact, it is a mixture of topology and metric.

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1. Introduction

Spatial reasoning is a major requirement for a comprehensive GIS (Frank 1991) because it offers users new spatial information, which has not been explicitly recorded and which is otherwise not immediately available in the form of raw data (Egenhofer 1994). To facilitate such reasoning, spatial relations between entities have to be established. In this sense, some researchers even argue that spatial relations between spatial entities are as important as the entities themselves.

Over the past two decades, research has been conducted on how to apply fundamental mathematical theories for modelling and describing spatial relations (Peuquet 1986, Jungert 1988, Chang *et al.* 1989, Lee and Hsu 1990, Kainz 1990, Egenhofer and Franzosa 1991, Egenhofer and Al-Taha 1992, Hadzilacos and Tryfona 1992, Smith and Park 1992, Cui *et al.* 1993). Here, no attempt has been made by the authors to discuss the various types of spatial relations. Instead, this paper concentrates on topological relations because 'topological properties are the most fundamental, compared to Euclidean, metric and vector spaces' (Egenhofer 1991).

Topological relations are those which are invariant under topological transformations. That is, they are preserved if the entities are translated, rotated or scaled (Egenhofer 1991). A formalization of topological relations has been investigated in later 1980s based on point-set topology (Guting 1988, Pullar 1988, Egenhofer and Franzosa 1991). The results of such a formalization are the so-called 4-intersection (Egenhofer and Franzosa 1991) and 9-intersection models (Egenhofer and Hering 1991). Indeed, the former is the foundation model based on intersections and the latter is an extension of the former (Egenhofer *et al.* 1993). In these models, the topological relations between two entities A and B are defined in terms of the intersections of A's interior, boundary and exterior with B's interior, boundary and exterior. The exterior of an entity is then represented by its complement.

The 9-intersection model has been regarded as the most comprehensive model for topological spatial relations so far. Analysis of this model has been made by researchers (Egenhofer 1991, Egenhofer *et al.* 1993, 1994, 1995, Clementini *et al.* 1994). This model has been used or extended for examining the possible topological relations between areas in discrete space (Egenhofer and Sharma 1993, Winter 1995), for modelling conceptual neighborhoods of topological line-area relations (Egenhofer and Mark 1995), for grouping the very large number of different topological relations for point, line and area features into smaller sets of meaningful relations (Clementini *et al.* 1993), for describing the directional relations between arbitrary shapes and flow direction relations (Abdelmoty and Williams 1994, Papadias and Theodoridis 1997), for deriving the composition of two binary topological relations (Egenhofer 1991), for describing changes of topological relations by introducing a Closest-Topological-Relationship-Graph and the concept of a topological distance (Egenhofer and Al-Taha 1992), for analysing the distribution of topological relations in geographical data sets (Florence and Egenhofer 1996), as well as for formalizing the spatio-temporal relations between the parent-child parcels during the process of land subdivision (Chang and Chen 1997). These investigations have contributed significantly to the development of state of the art, spatial data models and spatial query functionality (Egenhofer and Mark 1995, Mark *et al.* 1995, Papadis and Theodoridis 1997).

However, as will be discussed in §2, the extended 9-intersection model has two types of imperfections in theory. To improve this situation, this paper presents a modified model, called the Voronoi-based 9-intersection model, which results from the replacement of the complements of spatial entities by their Voronoi regions and represents a significantly extension of an earlier conference paper (Chen *et al.* 2000).

Following this introduction is a review and analysis of the existing 9-intersection model. In §2 the theoretical imperfections of this model are examined and the modified 9-intersection model, the Voronoi-based 9-intersection model, is presented in §3. The possible topological relations using the modified model are discussed in §4.

2. A critical examination of the 9-intersection model

In order to present an improved model in the next section, it seems logical to conduct a critical examination of the existing 9-intersection model in this section to see what imperfections it possesses and what improvements can be made.

2.1. *The development of the 9-intersection model*

In the early stages of research, the 4-intersection model of topological relations was proposed (Egenhofer and Franzona 1991) based on point-set topology. The principle is as follows: Suppose A and B are two sets representing two entities, then the topological spatial relations between A and B can be described by values of the 4-tuples as follows:

$$R_4(A,B)=\begin{pmatrix} A^0 \cap B^0 & A^0 \cap \partial B \\ \partial A \cap B^0 & \partial A \cap \partial B \end{pmatrix} \tag{1}$$

Where, ∂A is the boundary of A and A^0 is the interior of A and the annotation for B is the same. For example, if A and B are disjoint, then the values for these 4-tuples are $[-\phi, -\phi, -\phi, -\phi]$. For another example, if A and B are overlapping, then the 4 values becomes $[-, -\phi, -\phi, -\phi]$. Here ϕ means empty and '$-\phi$' means non-empty. There are two possible values for each of the 4 intersections, one can distinguish 2^4. Eight relations can be identified between two areas as shown in figure 2(*a*). These relations are mutually exclusive and form a partition of the set of all relations such as 'disjoint', 'overlap', 'touch', 'equals', 'cover', 'in', etc. This is the first model of its kind and it lays a solid foundation for further research of topological spatial relations.

However, as pointed out by Clementini *et al.* (1993), there are some cases where some confusion may be caused by this 4-intersection model. Indeed, the limitations of the 4-intersection model have been extensively discussed by (Egenhofer *et al.* 1993). Figure 1 illustrates only some examples. For this reason, Egenhofer and his

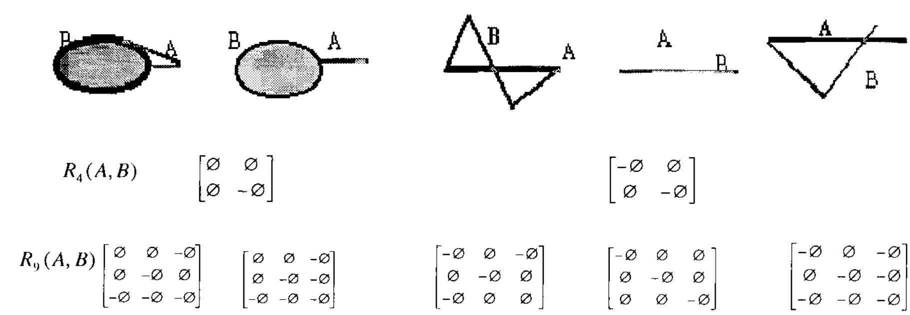


Figure 1. Improvement of the 9-intersection over the 4-intersection model. (For the first two relations, the 4-intersection model fails to distinguish between them, however in the 9-intersection model, the distinction is clear. The situation is similar for the last 3 relations).

collaborators (Egenhofer and Herring 1991) made an extension to the 4-intersection model, leading to a new model called the 9-intersection model, as follows:

$$R_9(A,B)=\begin{pmatrix} A^0\cap B^0 & A^0\cap\partial B & A^0\cap B^- \\ \partial A\cap B^0 & \partial A\cap\partial B & \partial A\cap B^- \\ A^-\cap B^0 & A^-\cap\partial B & A^-\cap B^- \end{pmatrix} \qquad (2)$$

Here, A^0, ∂A and A^- mean the interior boundary and exterior of A, respectively. The annotation for B is the same. In this model, the exterior of A is normally defined as the complement (other definitions of exterior would not fulfill the setting of the 9-intersection model). It is clear, as shown in figure 1, some of the limitations associated with the 4-intersection model are overcome in this model.

The 9-intersection model has been the most popular mathematical framework for formalizing topological spatial relations. This model considers whether the value (i.e. empty or non-empty) of the 9-intersections, a range of binary topological relations, can be identified (Egenhofer and Sharma 1993). For instance, eight relations as shown in figure 2(*a*) can be identified between two areas in R^2, i.e. *disjoint*, *meet*, *equal*, *inside*, *contains*, *covers*, *covered-by* and *overlap* (Egenhofer and Sharma 1993). Similarly, as shown in the rest of figure 2, topological relations between area-line, area-point, line-line, line-point as well as point-point can be defined (Egenhofer 1993, Egenhofer *et al.* 1993, Sun *et al.* 1993).

These relations are mutually exclusive. That is, only one of them holds at any time for a particular configuration.

2.2. *A critical comparison of the 4- and 9-intersection models*

It would then be of interest to have a critical comparison of the 4- and 9-intersection models. Indeed, such a comparison has already been made by Egenhofer and his collaborators and some of their conclusions are directly quoted (Egenhofer *et al.* 1993) as follows:

(a) With the 9-intersection model, the same set of area-area relations can be found as the 4-intersection model. No additional relations, due to the consideration of exterior intersections, are possible.
(b) As expected, the 9-intersection model reveals the same number of line-line relations in IR^1 as the 4-intersection model. However, in IR^2, the 9-intersection identifies another 25 relations for relations between two simple lines (i.e. line with exactly 2 end points). Another 21 relations are found if the lines can be branched so that they have more than two end points.
(c) With the 9-intersection model, 19 topological relations between a simple line and a region in IR^2 can be found and a 20th configuration is possible if the line is branched.

Egenhofer *et al.* (1993) have also pointed out when the addition of exterior matters. They found:

(a) If the two entities are simply connected, their boundaries form Jordan Curves and the entities have co-dimension 0, then the same eight topological relations can be realized as between two areas in IR^2.

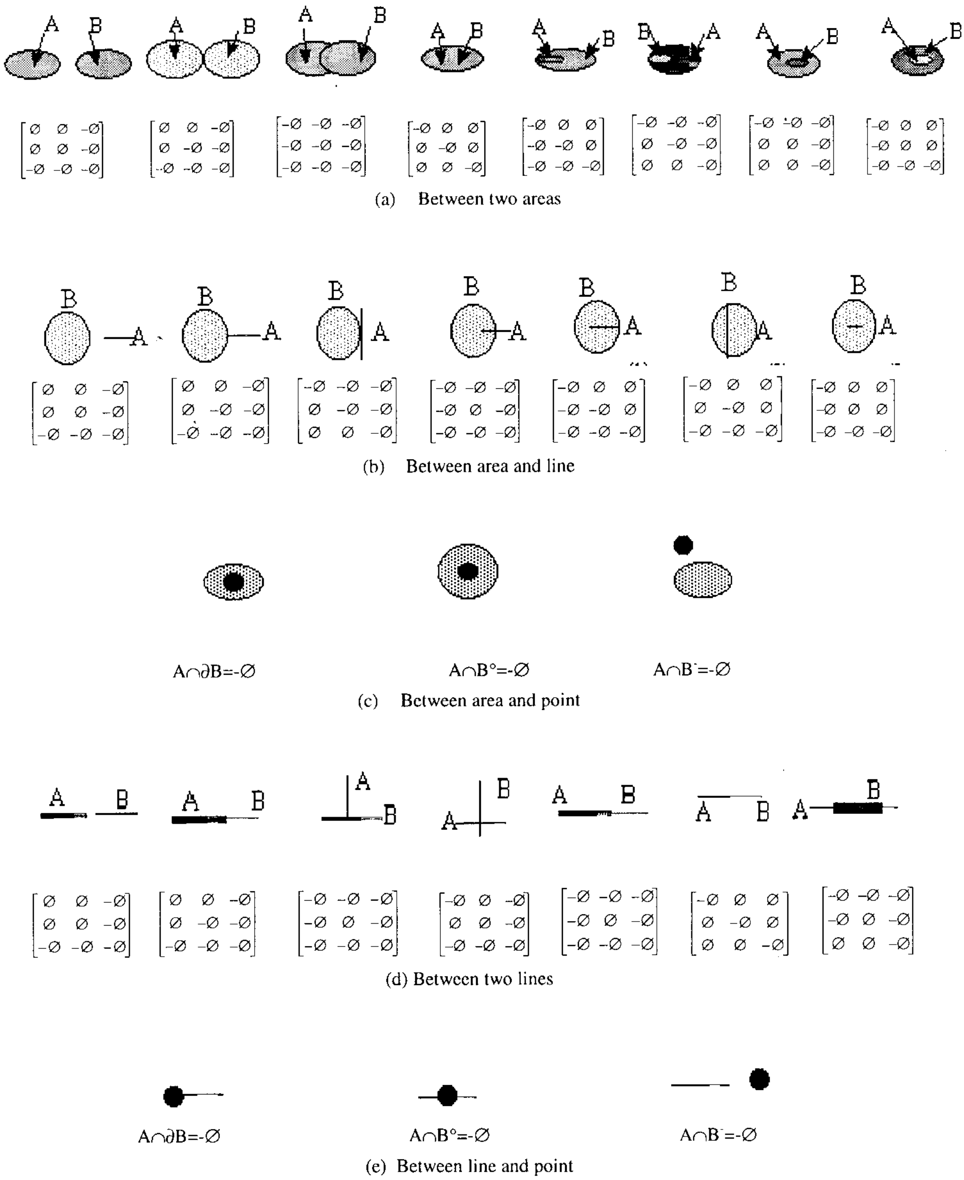


Figure 2. Topological spatial relations defined by the 9-intersection model.

(b) If the two entity are simply connected, each boundary forms a separation, and the entities have co-dimension 0, then the same eight topological relations can be realized as between two lines in IR^1

2.3. *Imperfection of the 9-intersection model*

The discussions presented in §2.2 reveal some basic facts about the relations between the 4- and 9-intersection models. It says that the number of relations existing between entities depends on the dimensions of the space with respect to the dimensions of the entities and on the topological properties of the entities embedded in that space (Egenhofer and Sharma 1993). In other words, if two entities are simply connected, they have co-dimension 0 and each boundary forms a separation between interior and exterior, then the 4- and 9-intersection models are equivalent. One would then wonder why this happens. A close examination of the definition of the 9-intersection model reveals that this is due to the definition of exterior as complement. The problem is illustrated in figure 3.

In figure 3, A and B are two areas (regions); C is a larger area containing both A and B and can be considered as the universe of the study area. It is clear that the following equation holds:

$$\begin{cases} C = A^0 + \partial A + A^- \\ C = B^0 + \partial B + B^- \end{cases} \tag{3}$$

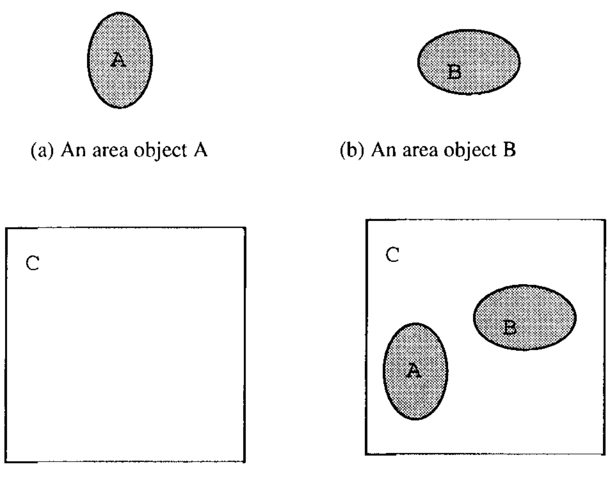
or

$$\begin{cases} A^- = C - (A^0 + \partial A) \\ B^- = C - (B^0 + \partial B) \end{cases} \tag{4}$$

By substituting (4) into (2), the following model is obtained:

$$R_9(A, B) = \begin{pmatrix} A^0 \cap B^0 & A^0 \cap \partial B & A^0 \cap (C - B^0 - \partial B) \\ \partial A \cap B^0 & \partial A \cap \partial B & \partial A \cap (C - B^\circ - \partial B) \\ (C - A^0 - \partial A) \cap B^0 & (C - A^0 - \partial A) \cap \partial B & (C - A^0 - \partial A) \cap (C - B^0 - \partial B) \end{pmatrix} \tag{5}$$

It means that, given an entity A, its complement is completely determined by a linear function of C and itself. As C is a constant space, therefore, the exterior (defined as the complement), boundary and interior of A (or B) are linearly dependent.



(a) An area object A (b) An area object B

(c) An area space C (d) C containing A and B

Figure 3. The definition of the exterior of an entity A (or B) as its complement in the 9-intersection model which causes the exterior of A (or B) to be completely determined by C and itself, i.e. $A^- = C - A = C - (A^0 + \delta A)$.

Therefore, there is a one-degree redundancy in this model. As a result, no additional information can be provided by extended 9-intersection model. This explains why the result of the 9-intersection model is equivalent to the 4-intersection model in this case (Egenhofer *et al.* 1993). In other words, the extension from 4-intersection to 9-intersection is of no help under these conditions.

Then one may wonder why additional topological relations could be distinguished by the 9-intersection model, if the co-dimension constraint is relaxed such that one or both entities can be embedded in a higher-dimensional space. The question could be asked in an alternative way, i.e. why additional relations can be distinguished if a line is involved (i.e. line-line and line-area).

To answer this query, one should examine the definitions of the three basic components of a line, i.e. the interior, boundary and the exterior, in both 1- and 2-dimensional space. In 1-dimensional space IR^1, the boundary of a line is defined by the two end points. The boundary separates its interior from its exterior. However, in a 2-dimensional space IR^2, a line's boundary doesn't separate the interior from its exterior any more, as pointed out by Egenhofer *et al.* (1993). In other words, the adoption of a definition in IR^1 to IR^2 causes the change of a basic topological property, i.e. Proposition 3.4 in Egenhofer's original paper (Egenhofer and Franzosa 1991, p. 166). Although some remedies have been made by Egenhofer and his collaborators (Egenhofer *et al.* 1993), this adoption causes at least inconsistency for the property of the line's boundary.

Due to this change of the topological properties of the boundary of an entity, the linear dependency between interior, boundary and exterior (as complement in the 9-intersection model) as expressed by equations (3) and (4) disappears so that additional topological relations can be distinguished by the 9-intersection model in comparison with the 4-intersection model. In other words, the distinction of additional relations by the 9-intersection is purely due to the simple adoption of 1-dimensional definitions of the interior, boundary and exterior of lines to a 2-dimensional space.

The questions arising are: 'is there any easy solution to remedy these two type of imperfections in the theoretical background of the 9-intersection model?' The answer to this question is yes and it will be presented in the next section.

3. The Voronoi-based 9-intersection model: an improved solution

After the imperfections have been pointed out, it seems logic to find out a solution if possible.

3.1. *The need of an alternative to the exterior of an entity*

As discussed in the previous section, there are two imperfections of the 9-intersection model The first one is that it defines the exterior of an entity as its complement so that the interior, boundary and exterior are linearly dependent. This causes the 9-intersection model to not work as effectively as it should. To remedy this imperfection, the exterior of entity should be defined as something else instead of its complement.

The second imperfection is that it adopts the 1-dimensional definition of a line to IR^2 so as to cause an inconsistency of a line's topological property in IR^1 and IR^2, i.e. the boundary of an entity doesn't separate its interior from its exterior in IR^2. This can be remedied by making use of a entity's other components rather than its exterior, if feasible.

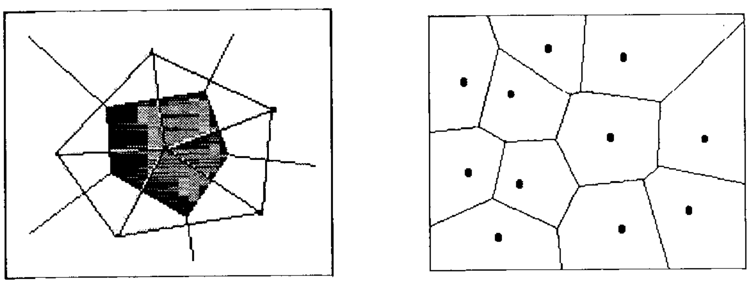
It has been found that, if the Voronoi diagram of an entity (as shown in figure 4), is used to replace its exterior in the 9-intersection model, the theoretical imperfections of this model could be improved.

3.2. *The Voronoi region as an alternative to the exterior of an entity*

Then one may wonder why the Voronoi region of an entity is such an appropriate alternative for its exterior in the 9-intersection model. To answer this query, a discussion of the definition and properties of a Voronoi region is needed.

A Voronoi region is a 'region of influence' or 'spatial proximity' for each spatial data point. All these Voronoi regions together will form a pattern of packed convex polygons covering the whole plane (neither any gap nor any overlap). This result of tessellation is called a Voronoi diagram.

Voronoi diagram is essentially 'a partition of the 2-D plane into N polygonal regions, each of which is associated with a given point. The region associated with a point is the locus of points closer to that point than to any other given point' (Lee and Drysdale 1981). The polygonal region associated with a point is normally called the 'Voronoi region' (or Thiessen polygon) of that point and it is formed by perpendicular bisectors of the edges of its surrounding triangles (figure 4). Figure 4 shows only the case of point entities. In fact, it is also possible to define and compute the Voronoi-diagrams of any spatial entity such as points, lines and areas (figure 5). Indeed, the definition of Voronoi-region can also be modified to cover complex entities. Figure 6 illustrates such a modified definition. More detailed discussion of



(a) Voronoi region of a point (b) Voronoi diagram

Figure 4. Voronoi region and Voronoi diagram.

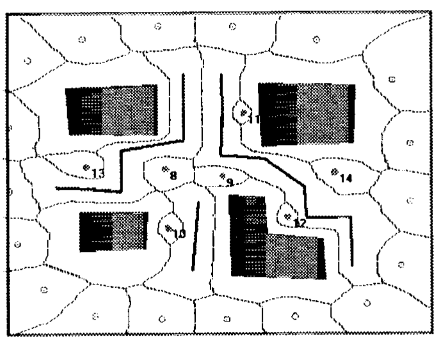
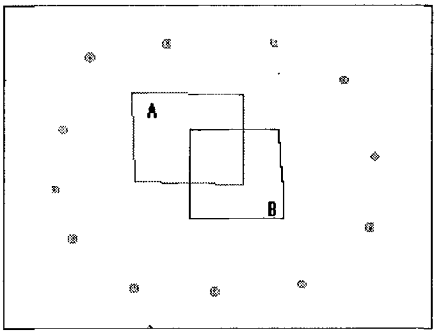
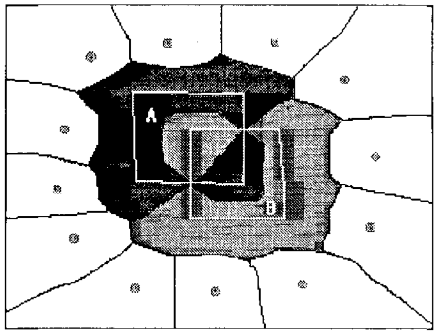


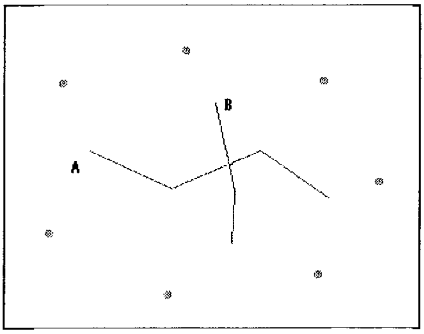
Figure 5. Voronoi diagram of point, line and area entities.



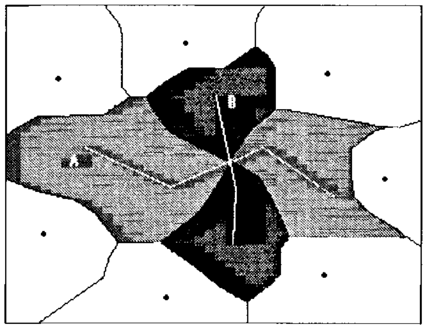
(a) Two overlapping areas and a few points



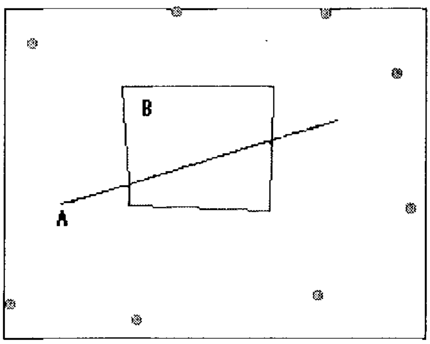
(b) The Voronoi diagram of (a): the dark and grey areas are the Voronoi-diagram of A and B, respectively.



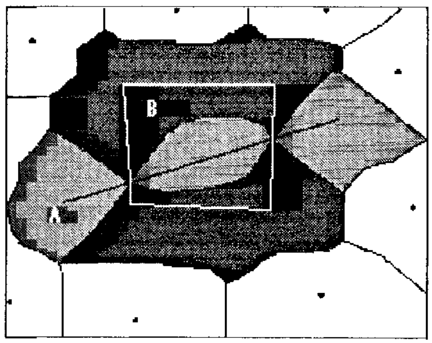
(c) Two intersecting lines and a few points



(d) The Voronoi diagram of (c): the dark and grey areas are the Voronoi-diagram of B and A, respectively.



(e) A line intersecting an area with a few points



(f) The Voronoi diagram of (c): the dark and grey areas are the Voronoi-diagram of B and A, respectively.

Figure 6. Voronoi diagram of complex spatial entities.

how to compute Voronoi-diagrams for complex entities are described by Li *et al.* (1999).

The properties of the Voronoi diagram have been studied by many researchers (Gold, 1989, 1991,1992, Wright and Goodchild 1997), and it has found many applications (Yang and Gold 1994, Gold *et al.* 1996, Edwards *et al.* 1996, Hu and Chen 1996, Chen and Cui 1997). Algorithms for the computation of Voronoi diagrams in vector mode have been summarised by Aurenhammer (1991) and Ohya *et al.* (1984). An algorithm in raster mode has been developed by Li *et al.* (1999a), where the definition and computation of Voronoi diagrams for complex spatial entities have also been addressed.

In IR^2, Voronoi regions of all entities together form a contiguous and non-overlapping tessellation of space. The Voronoi region of an entity is determined by partitioning space with other neighbouring entities and therefore is a function of the locations and shapes of its neighbours and itself. Therefore, the use of the Voronoi region of an entity to replace its complement (as exterior) in the 9-intersection model avoids the linear dependency expressed in equations (3) and (4). In the case of a line, the use of the Voronoi-region of an entity to replace its exterior avoids the inconsistency of a line's topological property in IR^1 and IR^2. This is because the Voronoi-region of a line has a similar function of the line's exterior but there is no such requirement that a line's boundary separates its interior from its Voronoi-region. It means that the Voronoi-region of an entity is really the appropriate replacement for the exterior in the 9-intersection model.

3.3. *The Voronoi-based 9-intersection model*

By replacing the complement of an entity with its Voronoi region, a Voronoi-based 9-Intersection (V9I for brevity) framework can be formulated as follows:

$$R_{v9}(A,B)=\begin{pmatrix} A^0\cap B^0 & A^0\cap\partial B & A^0\cap B^v \\ \partial A\cap B^0 & \partial A\cap\partial B & A^0\cap B^v \\ A^v\cap B^0 & A^v\cap\partial B & A^v\cap B^v \end{pmatrix} \quad (6)$$

where A^v is Entity *A*'s Voronoi region and B^v is Entity *B*'s Voronoi region.

4. Topological relations with V9I

After proposing the model, it is necessary (*a*) to examine the spatial topological relations defined by this model, and (*b*) to describe what additional relations this new model may be able to distinguish.

4.1. *Topological relations of simple entities with V9I*

Topological relations between point, line and area entities can be formalized with the new model, including relations between area-area, line-area, area-point, line-line, line-point and point-point entities. The results are listed in table 1. A diagrammatic representation of these relations is given in figures 7–12.

As can be seen from figure 7, among the thirteen topologically distinct relations

Table 1. Distinguished topological relations using V9I.

Cases		Number
AA	Area/Area	13
LL	Line/Line	8
LA	Line/Area	13
PP	Point/Point	3
PL	Point/Line	4
PA	Point/Area	5

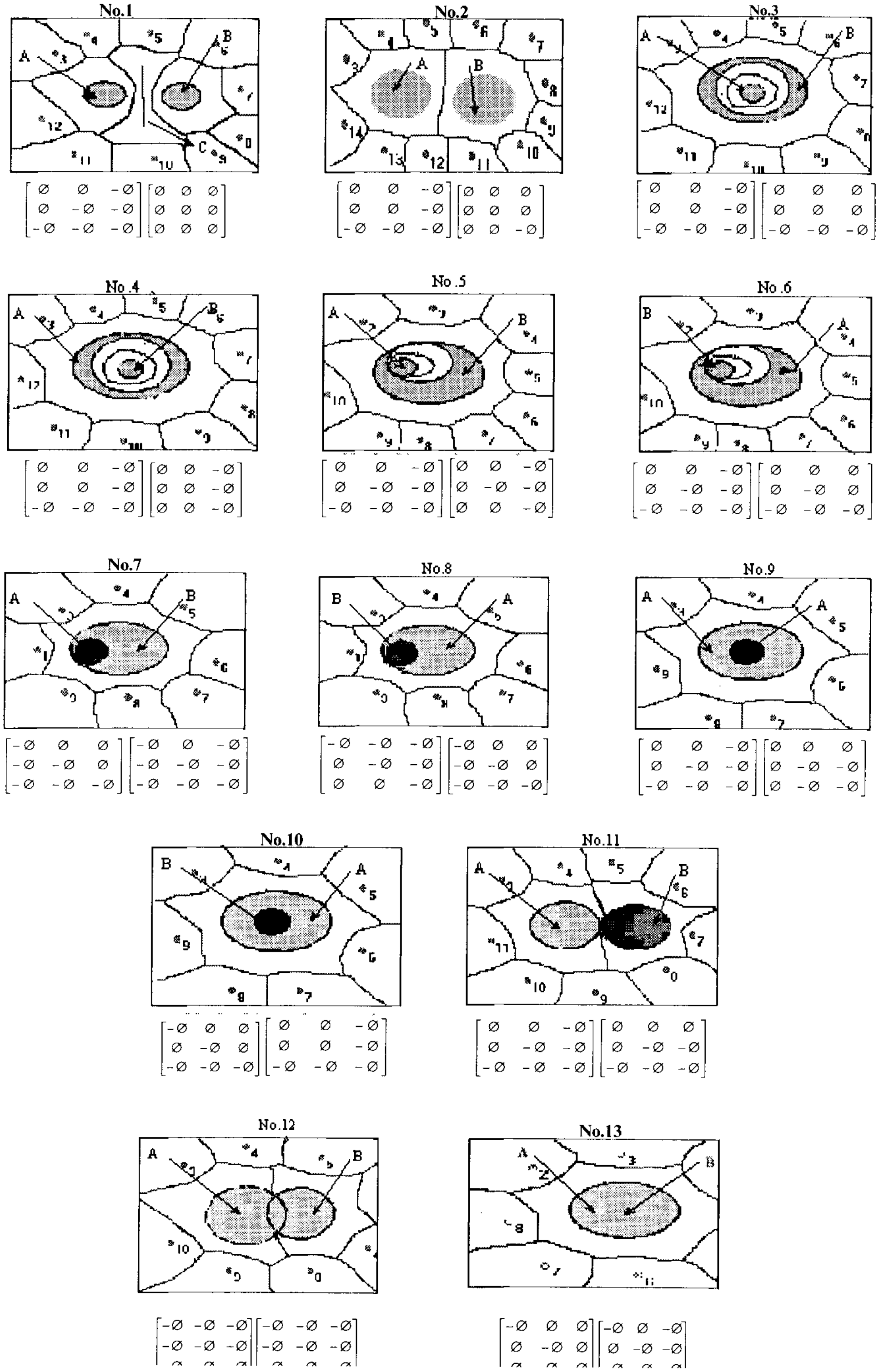


Figure 7. Topological relations for area-area relations distinguished by the V91 model (the 9-intersections are shown on the left and the V9-intersection on the right).

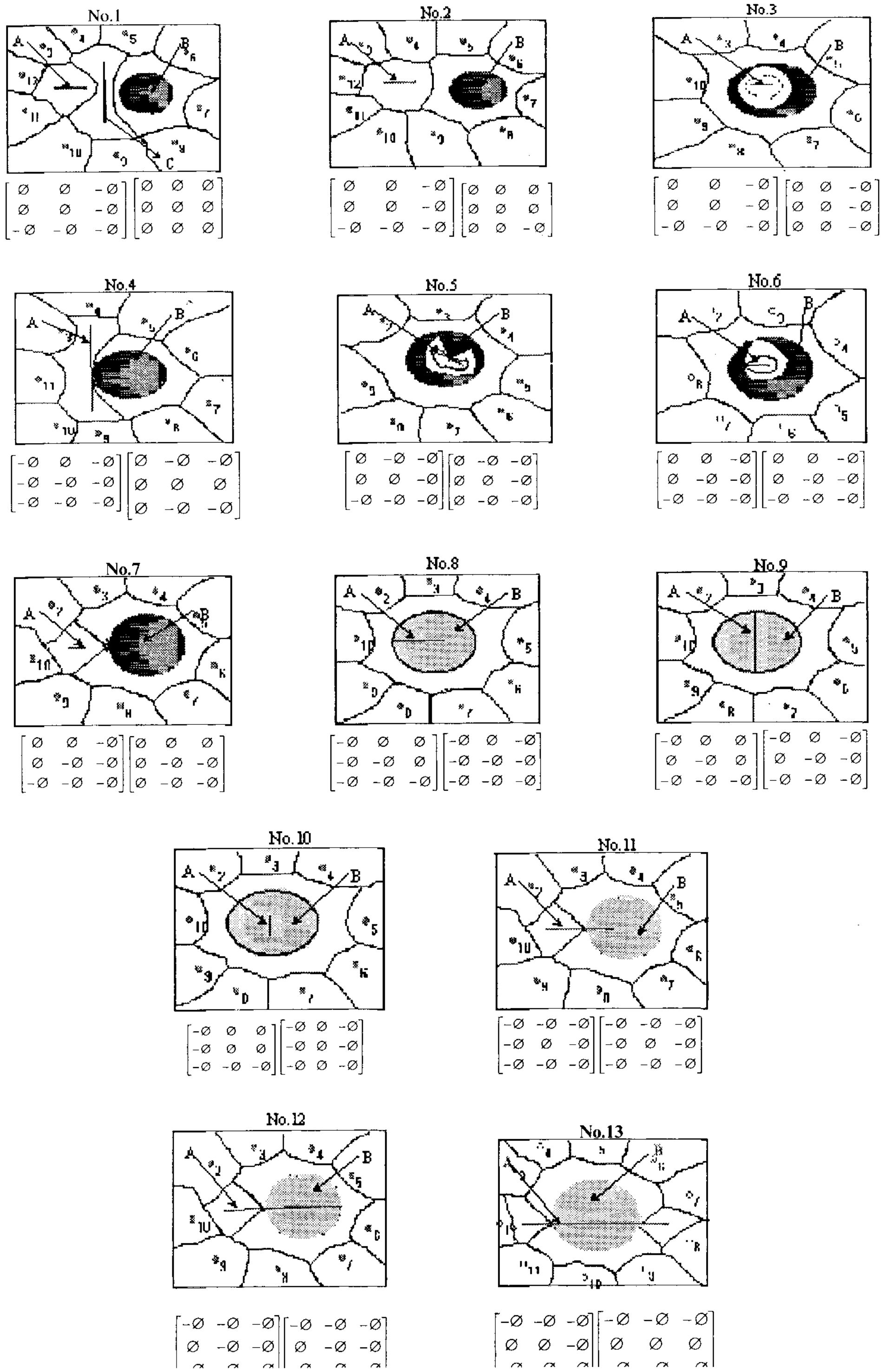


Figure 8. Topological relations for area-line relations distinguished by the V91 model (the 9-intersections are shown on the left and the V9-intersection on the right).

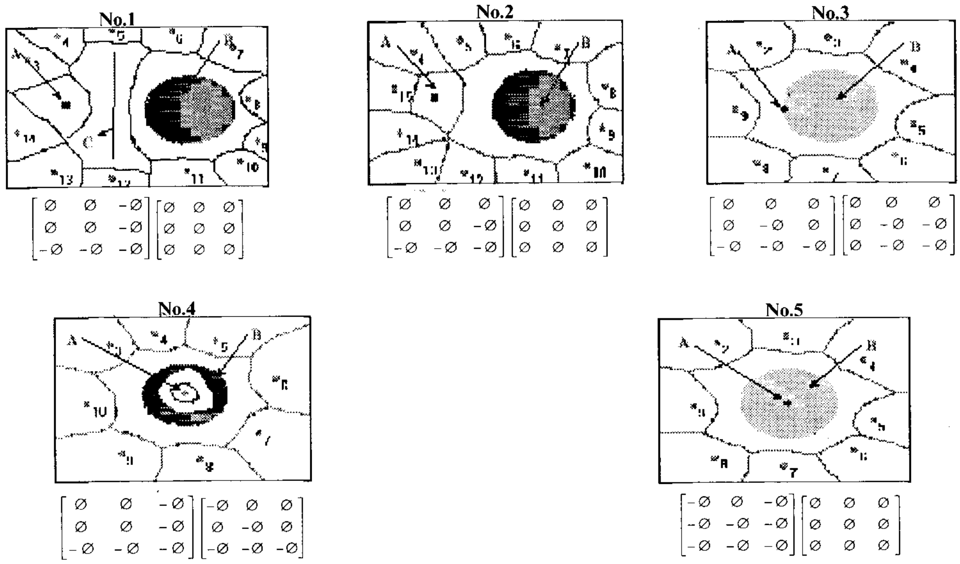


Figure 9. Topological relations for area-points distinguished by the V91 model (the 9-intersections are shown on the left and the V9-intersection on the right).

between two areas distinguished by the V9I, seven of them could not be distinguished using the original 9-intersection model. The advantage of V9I model is therefore clear.

4.2. *Topological relations of complex entities with V9I*

With the V9I, the distinction of more relations of spatial entities is also possible in the case of a complex entity (figure 13).

One characteristic of this new model is that $\partial A \cap \partial B$ and $A^v \cap B^v$ would both take non-empty values when the boundary of entity A meets with that of entity B as their Voronoi regions would also meet according to the definition of the Voronoi diagram (as illustrated in figure 13(*a*)). In addition, the boundary of entity A meets with B^v and B's boundary meets with A^v. As a result, the relation *meeting* between area entities as defined by the Voronoi-based 9-intersection model is therefore quite different from that defined by the original 9-intersection model. For an entity with a hole (e.g. B as shown in figure 13(*b*)), if the boundary of the other entity A meets its inner boundary, A's Voronoi region intersects B's inner boundary, resulting in 4 non-empty elements as follows: $\partial A \cap \partial B = -\varnothing$, $A^v \cap \partial B = -\varnothing$, $A^v \cap B^v = -\varnothing$ and $\partial A \cap B^v = -\varnothing$. Moreover, if the whole body of A is contained in B's hole, it means that A's interior overlaps with the convex of B and $A^0 \cap B^v = -\varnothing$. These characteristics make the Voronoi-based 9-intersection model capable of distinguishing relations between complex entities with holes.

The example shown in figure 13(*b*) has the same original 9-intersections, but has different Voronoi-based 9-intersections than those in figure 13(*a*). The example illustrated in figure 13(*d*) is a *contained-by* relation which has the same 9-intersections as the *contains* relations shown in figure 13(*e*). The Voronoi regions touch and there is no intersection of boundaries and interiors between the two entities. However, the boundary and interior of the contained entity intersect with the Voronoi convex of the other entity. Another example, given by figures 13 (*f*–*g*), shows a line meeting a

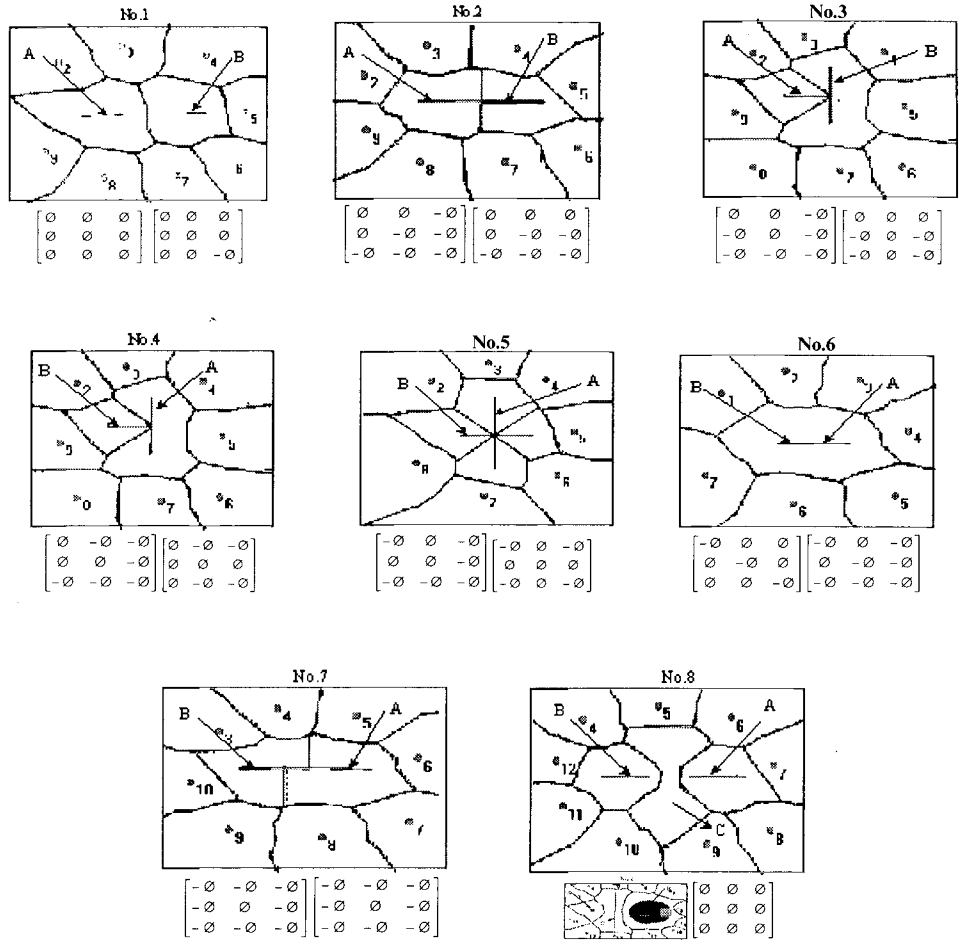


Figure 10. Topological relations for line-line relations distinguished by the V91 model.

homogeneously 2-dimensional and connected area B and the line falling into an area's hole.

It should be noted here that spatial entities with holes have also been addressed by Egenhofer *et al.* (1994) with an extension of the 9-intersection. However, no attempt is made here to address this extended model. Rather, this section simply illustrates what the new model, V9I, can do for complex spatial entities.

4.3. *High-resolution of disjoint relations: Beyond topological relations*

Another characteristic of this new model is that $A^v \cap B^v$ would be non-empty when two entities are adjacent, such as A and B in figure 14(*a*), because the Voronoi region of A shares the same boundary with that of B. However, when there is an entity C between A and B (figure 14(*b*)), their Voronoi regions are separated by that of C and $A^v \cap B^v$ is empty.

It means that the V9I model provides higher resolution for disjoint relations of spatial entities. Indeed K-order neighbour relations can be described by using such a model (Zhao *et al.* 1999). However, such high-resolution disjoint relations by this

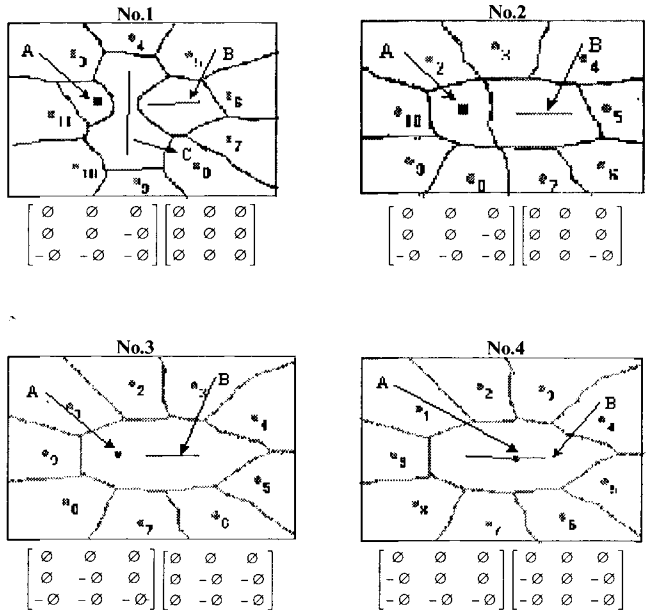


Figure 11. Topological relations for line-point relations distinguished by the V91 model.

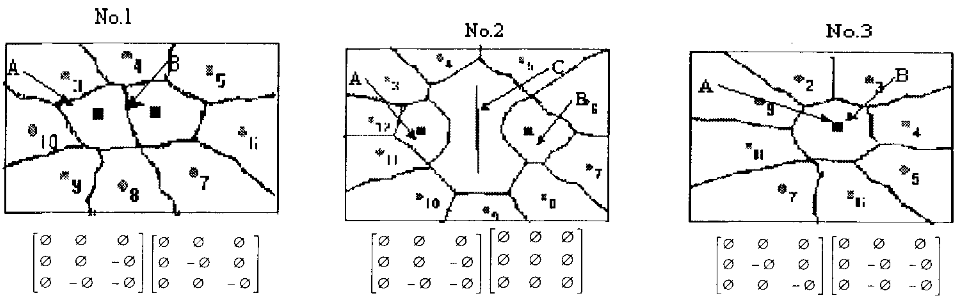


Figure 12. Topological relations for point-point relations distinguished by the V91 model.

model is not purely topological. It is, in fact, a mixture of metric and topological relations, as Voronoi regions may change under topological transformation. Such a change may result in another version of disjoint relations (figure 15). In figure 15(*a*), the disjoint relation of spatial entities *A* and *B* can be said as '1st order Voronoi-adjaceny' as their Voronoi regions are adjacent. However, after stretching, the relation becomes '2nd order Voronoi-adjacency' as the Voronoi regions of *A* and *B* are now separated by another Voronoi region.

5. Discussion and conclusions

In this paper, an examination of the development of intersection-based models for the formal presentation of topological relations has been made. The 4-intersection model developed by Egenhofer and Franzosa (1991) was the foundation model. Later, this model was extended by Egenhofer and Herring (1991) to 9-intersection.

It has been discussed that there are some imperfections in the theoretical aspect

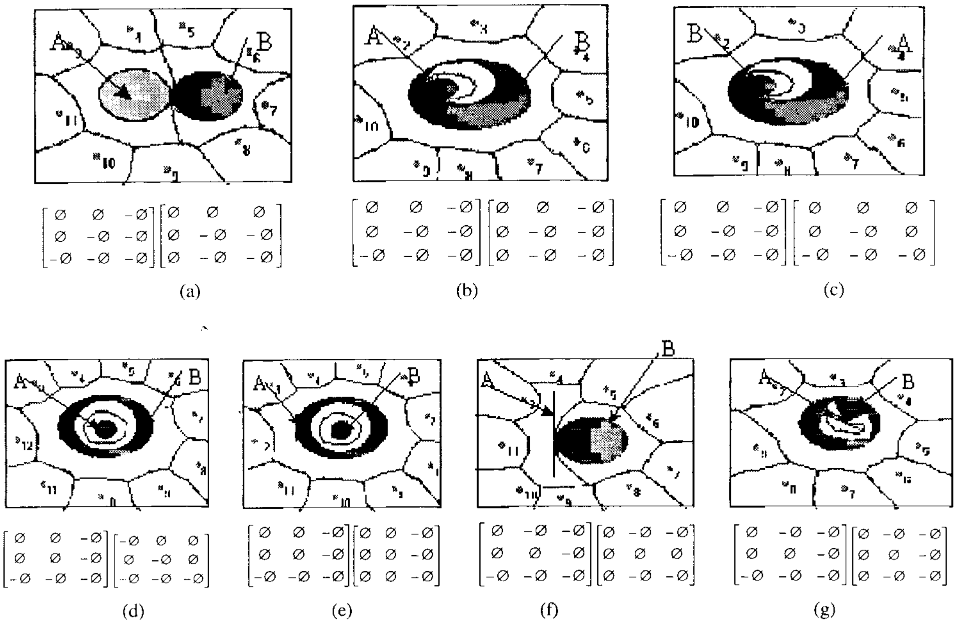


Figure 13. Resolution of complex spatial relations with the V91 model.

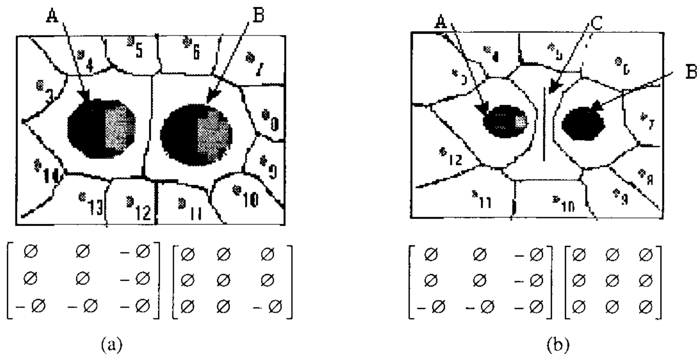
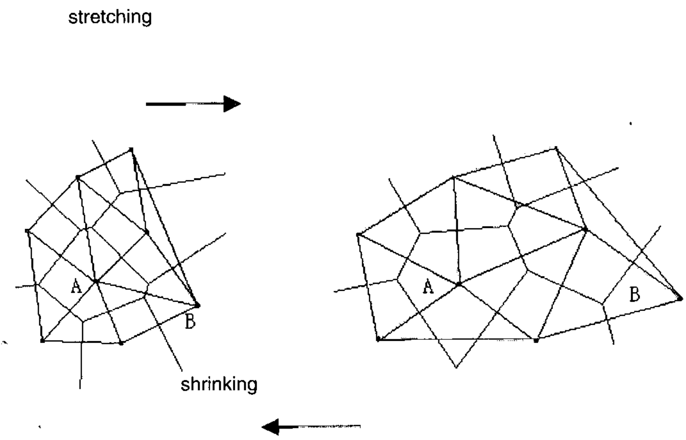


Figure 14. Resolution of disjoint relations with the V91 model.

associated with this extension. One of them is the linear dependency between the interior, boundary and complement (used for exterior) of an entity so that the extension of 4-intersection to 9-intersection is of no help, in terms of the number of relations. This has been confirmed by Egenhofer *et al.* (1993). Another imperfection is the adoption of 1-dimensional definitions of line's interior, boundary and exterior to a 2-dimensional space so that an inconsistency of a line's topological property in IR^1 and IR^2 has been caused (Li *et al.* 1999b).

It has been found that it is this change of conventional topological property that eliminates the linear dependency between the interior, boundary and complement (used for exterior) of an entity so that the additional relations can be distinguished by the 9-intersection model in comparison with the 4-intersection model. In other words, the distinction of additional relations by the 9-intersection is purely due to the imperfect definitions of the interior, boundary and exterior of lines in IR^2.



(a) 1st order Voronoi-adjacency

(b) 2nd order Voronoi-adjacency

Figure 15. Voronoi-adjacency relations between *A* and *B* described by V91. it is not invariant under topological transformation. Therefore, the high-resolution of disjoint relations by V91 is not purely topological. (Rather it is a mixture of topology and metric).

It has also been pointed out that the use of the Voronoi-region of an entity to replace its complement as its exterior in the 9-intersection model would solve the problem or would make this model more comprehensive. The Voronoi region of an entity is determined by partitioning space with other neighbouring entities and thus is a function of the locations and shapes of its neighbours and itself. Therefore, the use of the Voronoi region of an entity to replace its complement as exterior in the 9-intersection model avoids the linear dependency expressed in equations (3) and (4). In the case of a line, the use of the Voronoi-region of an entity to replace its exterior avoids the inconsistency of a line's topological property in IR^1 and IR^2. This is because the Voronoi-region of a line has a similar function of the line's exterior but there is no such requirement that a line's boundary separates its interior from its Voronoi-region. It means that the Voronoi-region of an entity is really the appropriate replacement for the exterior in the 9-intersection model. Therefore, a Voronoi-based 9-intersection is proposed in this paper.

In addition, with this Voronoi-based 9-intersection model (V9I), additional relations beyond simple topological relations can also be distinguished. One important type of relation the V9I can distinguish is the relation of complex spatial entities such as spatial entities with holes. Another of type is the high-resolution disjoint relation. This is very important because about 80% of spatial relations are disjoint relations (Florence and Egenhofer 1996). However, such high-resolution disjoint relations from this model are not purely topological. It is, in fact, a mixture of topology and metric, as Voronoi regions may change under topological transformation.

Spatial relations still have unsolved problems. More research in this area is desirable, but the authors hope that this paper makes a useful contribution to the research agenda.

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