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A comparative study of the accuracy of digital terrain models (DTMs) based on various data models

This paper describes a comparative study of the accuracies of digital terrain models (DTMs) derived from four different data models, namely, contour data only, contour data with additional feature-specific (F-S) data (peaks, pits, points along ridges, points along ravines, and points along break lines, etc.), square-grid data only, and square-grid data with additional F-S data. It has been found that:

(1) The accuracy of DTMs derived from (photogrammetrically measured) contour data, in terms of RMSE (root mean square error) or σ (standard deviation), is about 1/3 to 1/5 of the contour interval, depending on the characteristics of the terrain topography. However, if additional F-S data are included, these figures can be reduced by 40% to 60%.

(2) The accuracy of DTMs derived from gridded data plus F-S data is linearly related to the grid-interval. However, if F-S data are not included, the relationship becomes parabolic.

(3) The accuracy of a DTM derived from contour data with a vertical interval CI matches that from grid data with an interval of $(K \times CI \cot\alpha)$, where α is the mean slope angle of the area and K is a constant ranging from 1.5 to 2.0. However, if additional F-S data are included, then the value of K can be reduced. In this case, the value of K ranges from 1.0 to 1.5, depending on the characteristics of the terrain topography.

1. Introduction and background

The accuracy of digital terrain models (DTM) is of concern to both DTM users and DTM producers. It was an important research topic in ISPRS Commission III until 1988, when at the 16th ISPRS Congress Prof. K. Kubik delivered a report, claiming that basic problems in DTM accuracy estimation had been solved. Since then, DTM accuracy estimation has almost been absent from the ISPRS research agenda. However, Li (1990) argued that many basic problems in this topic still remain unsolved and existing mathematical models are either incapable of producing reliable results or impractical to use. Indeed, OEEPE has similar views to those of Li and has recently established a special working group for this topic. After a special meeting held on March 17th, 1992 at Basel, the working group decided to invite Prof. K. Kraus of the Technical University of Vienna to supervise the first experimental test. Therefore, it is still important to investigate accuracies of DTMs derived from various data models, especially to establish some relationship between them. This is the purpose of this study.

A brief review of the literature concerning the assessment of the accuracy of digital terrain models might be helpful in understanding the motivation and importance of this study. From this literature, it can be noted that, since the early 1970s, the research direction in digital terrain modelling has shifted from the development of interpolation techniques to the assessment and control of DTM quality (accuracy). There are numerous useful papers. Some examples of empirical studies are: Ackermann (1978, 1979), Ackermann and Stark (1985), Ley (1986), Torlegard et al. (1986), Day and Muller (1988), Tuladhar and Makarovic (1988), Li (1990, 1992) and Kumler (1990), etc. In spite of these efforts, only Ackermann (1979) managed to propose an empirical model for a particular case. On the other hand, quite a few mathematical models have been developed through theoretical analysis, for example, Makarovic (1972), Kubik and Botman (1976), Frederiksen (1981), Frederiksen et al. (1986). However, the evalua-

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tions by both Balce (1987) and Li (1990, 1993) indicate that no single one of these models is capable of producing reliable predictions. This means that a better understanding of error components in DTMs, both through theoretical analysis and empirical study, is still a matter of some urgency. This study is an attempt only through empirical study, but a theoretical analysis is reported elsewhere (Li, 1994).

However, not all possible data models are analysed in this study. Due to the fact that, through photogrammetric measurement, image matching, ground survey and contour digitisation, quite a number of data models would be yielded, such as regular grids, size-varying grids (through progressive sampling), strings along profiles, strings along contours, etc., it was decided to study only a few of the most important data models. Therefore, the square-grid and string along contours are the two models considered in this study. Also selectively sampled feature-specific (F-S) data (i.e. peaks, pits, points along ridges, points along ravines, and points along break lines, and faults, etc.) (for more information about F-S data, see for example, Chen, 1988 and Scarlatos, 1989, 1990) can be added to both contour data and grid data. Therefore, four data models result, i.e. contour data only, contour data with additional F-S data, grid data only and grid data with additional F-S data. These are the four data models to be considered in this study.

The main objective of this study is to carry out some investigations into three aspects of the larger problem, namely: (a) DTM accuracy improvement by adding F-S data to contour data, (b) DTM accuracy improvement by adding F-S data to gridded data, and (c) comparing the accuracy of DTMs derived from contour data (with and without additional F-S data) with that from grid data (also with and without additional F-S data).

In this paper, a description of test data, including the test area, the measured DTM source data and check points, is given followed by a description of the test procedure. Then the accuracies of the DTMs derived from contour data both with and without additional F-S data are presented and compared. Next the accuracies of the DTMs derived from square-grid data both with and without additional F-S data are described and compared; and finally a comparative analysis of the accuracies of the DTMs derived from both contour and grid data (both with and without additional F-S data) is carried out.

2. Test data

In this section, a brief description of the test area, DTM source data and check points will be presented.

The test areas used in this study are three of those used for the ISPRS DTM test which was conducted by Working Group 3 of Commission III (Torlegard et al., 1986). A set of photogrammetrically measured contour data, a set of square-grid data and a set of F-S data for each of these three areas were kindly made available to the author through the courtesy of Prof. H. Ebner and Dr. W. Reinhardt at the Technical University of Munich (Germany). These areas are known as Uppland, Sohnstetten and Spitze. The descriptions of these three test areas have been given by Torlegard et al. (1986) and Li (1992). A brief summary is given in Table 1. The contour plots of these areas are shown in Fig. 1, onto which the F-S data are superimposed. The Uppland area is relatively flat, with a few mounds. In the Sohnstetten area, a valley runs through the middle of the area so most of the F-S points are along edges and ravines. In the Spitze area, a road junction cuts through the right side of the area so that the F-S points are those along the break lines caused by these roads.

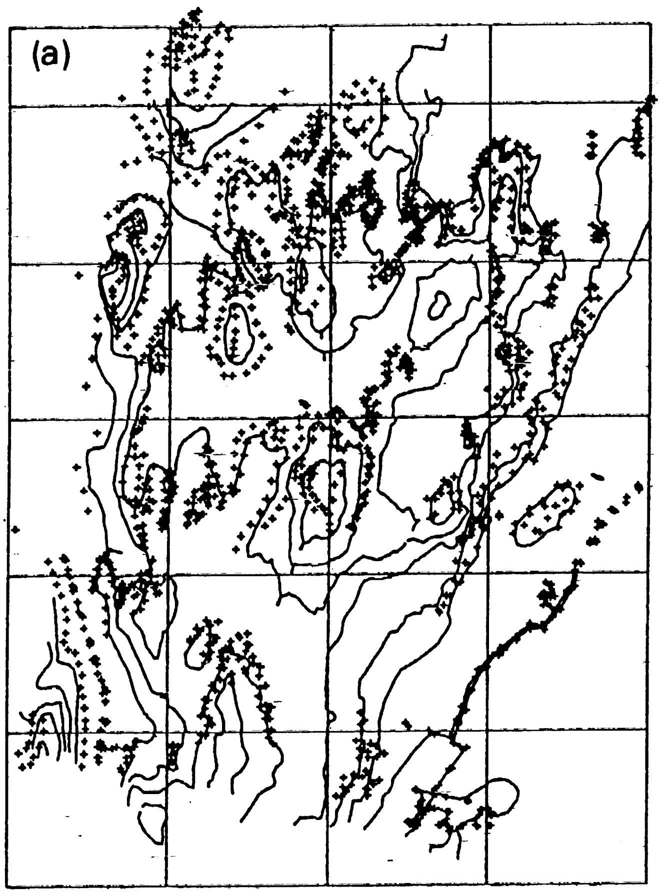
These data sets were measured on a Zeiss Oberkochen Planicomp C-100 analytical plotter. Some information about the contour data and the grid data is given in Table 2. Detailed information about these grids is given by Li (1992). These contour data can be regarded as being very dense along contour lines in comparison to their corresponding average planimetric intervals. Therefore, the contour interval (CI) is the most significant parameter of a contour data set.

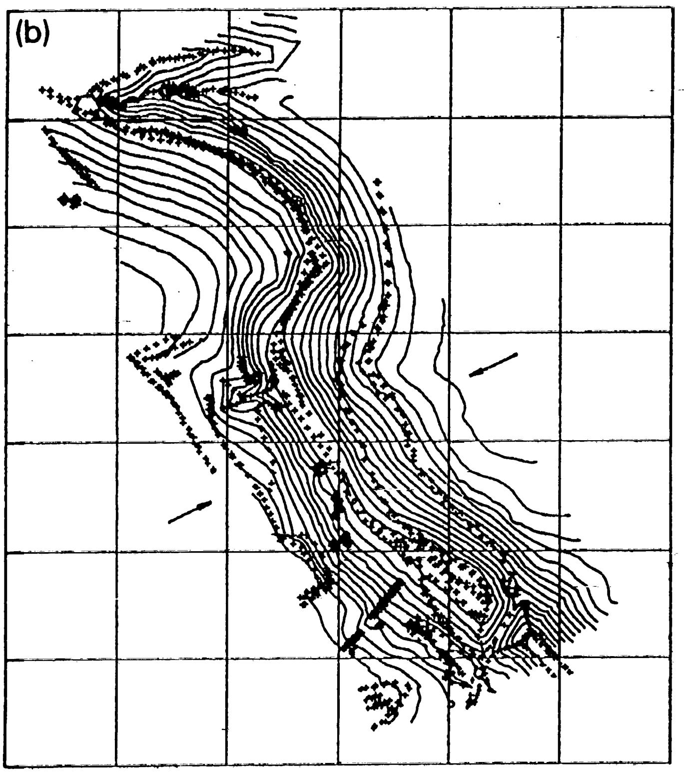
TABLE 1

Description of test data

Test area	Description	Height range (m)	Average slope (°)
Uppland	Farmland and forest	7– 53	6
Sohnstetten	Hills of moderate height	538–647	15
Spitze	Smooth terrain	202–242	7

The check points used in this test were measured from much larger-scale photographs in the Royal Institute of Technology (the ISPRS DTM test centre) in Stockholm and were kindly made available to the author through the courtesy of Prof. K. Torlegard and Dr. M.X. Li. More detailed information about these check points has also been





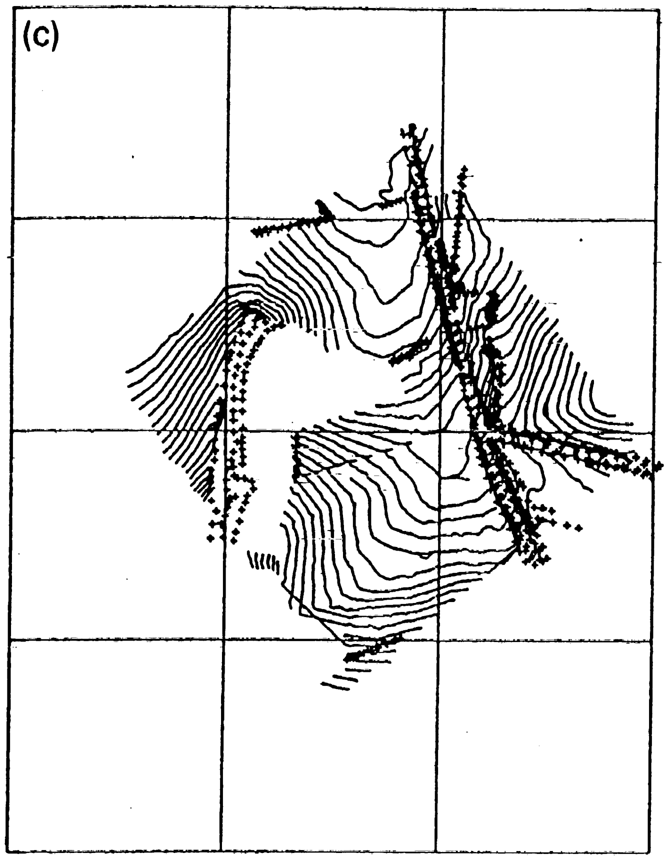


Figure 1. Contour plots of test areas (photogrammetrically measured), superimposed with feature-specific points: (a) for the Uppland area ($CI = 5$ m); (b) for the Sohnstetten area ($CI = 5$ m); (c) for the Spitze area ($CI = 1$ m), where the large blank area was not measured due to difficulties.

TABLE 2

Data sets for testing

Parameters	Uppland	Sohnstetten	Spitze
Photo scale	1:30,000	1:10,000	1:4,000
Flying height (H)	4,500 m	1,500 m	600 m
Grid interval	40 m	20 m	10 m
Grid data accuracy	±0.67 m	±0.16 m	±0.08 m
Contour interval (CI)	5 m	5 m	1 m
Ave. P. CI [a]	48 m	9 m	8 m
Contour point interval	10.4–22.5 m	3.7–19.8 m	5.4–9.2 m
Contour data accuracy	±1.35 m	±0.45 m	±0.18 m

[a] Average planimetric contour interval (which is equal to $CI \cot\alpha$, where α is the everage slope angle); accuracy here is in terms of RMSE.

TABLE 3

Information on check points

Test area	Photo scale	Flying height (m)	Number of points	RMSE (m)	Maximum errors errors (m)
Uppland	1:6,000	900	2,314	±0.090	0.20
Sohnstetten	1:5,000	750	1,892	±0.054	0.07
Spitze	1:1,500	230	2,115	±0.025	0.05

given by Törlegard et al. (1986) and Li (1992). A brief summary is given in Table 3.

3. Test procedure

A triangulation-based DTM package — PANACEA, which was developed by McCullagh (1988) of Nottingham University — was used for this study. The popular Delaunay triangulation algorithm (see McCullagh and Ross, 1980) is implemented in this package. Individual contours are treated as break lines; after the triangulation process by the Delaunay algorithm, an additional sorting process is applied to ensure that no triangles can cross a contour and at most two points can be selected from a contour line for each triangle.

With this package, the input data (either contour data or grid data) were triangulated, then a continuous surface comprising a series of contiguous linear facets was constructed from the triangular network. Finally, the DTM points were interpolated from the triangular facets. By comparing the DTM points and check points, a set of height residuals could be obtained for each test area. Accuracy estimates were made from this set of residuals. In this study, the RMSE (root mean square error) as well as the mean (μ) and standard deviation (σ) were used. The reliability of these statistics is affected by the characteristics of the set of check points (Li, 1991). However, in this study such an effect can be neglected because the accuracy of the check points is much higher than that of the DTM source data and a large sample of check points has been used in each area.

4. Improvement in the accuracy of the DTMs derived from contour data with additional feature-specific data

The aim of this investigation was to determine the relationship between the vertical interval of the contours and the accuracy of the resulting DTMs and how much improvement in the accuracy of the final DTMs can be achieved if F-S data are included.

The accuracies of the DTMs derived from contour data only for these three study areas are shown in Table 4, where, $+E_{\max}$ and $-E_{\max}$ represent the positive and negative maximum errors, respectively.

If these values are expressed in terms of "per mil of H" (i.e. per mille of flying height), then they are from about 0.3 to 0.6. In two areas, the values are very much greater than 0.3 per mil of H, which is the expected accuracy of dynamically measured contour data. Therefore, a large error component appears to come from the loss in fidelity of the terrain topography which is represented selectively by the measured contour lines.

Since, as discussed previously, the contour interval (CI) is the most significant parameter of a contour data set, the values of the DTMs derived from contour data can also be expressed in terms of a fraction of CI, as an analogue to the conventional expression for map accuracy. The results obtained from this test, as shown in Table 4, are within a range from $CI/3$ to $CI/5$, approximately. Of course, if the contour data are digitized from an existing map, the accuracy of the resulting DTMs would be lower than this value.

When considering the error contribution from the source data, the following empirical model, which is an analogue to the conventional map accuracy specification, is used for further analysis:

$$\sigma_{\mathrm{DTM}}^2 = \frac{\sigma_{\mathrm{DCD}}^2}{C} + \left(\frac{CI}{K}\right)^2 \tag{1}$$

where σ_{DCD}^2 denotes the error variance of the measured digital contour data; CI the contour interval; K and C are constants; and σ_{DTM}^2 denotes the accuracy of the resulting DTM in terms of variance.

TABLE 4

DTM accuracy (σ-value) improvement by adding feature-specific data to the contour data

Parameters	Uppland		Sohnstetten		Spitze	
	with F-S	without F-S	with F-S	without F-S	with F-S	without F-S
RMSE (m)	0.93	1.74	0.35	0.91	0.17	0.27
μ (m)	0.47	1.05	0.11	0.22	0.09	0.10
σ (m)	0.80	1.39	0.35	0.88	0.15	0.24
$+E_{max}$ (m)	3.25	5.91	1.73	4.52	0.75	0.94
$-E_{max}$ (m)	−5.18	−5.18	−2.48	−3.01	−0.95	−0.95
σ/H (‰)	0.18	0.31	0.23	0.59	0.25	0.40
CI/σ	6.25	3.60	4.29	5.68	6.67	4.17
K (eq. 1)	27.7	4.5	21.3	5.9	9.2	4.6
σ improvement	42.45%		60.23%		37.50%	

F-S denotes feature-specific data; σ improvement means the improvement in standard deviation values; K (eq.1) means the K values expressed in eq. 1 in the text; σ/H (‰) means the values of σ divided by H (the flying height at which the photographs were taken) in terms of per mil of H.

Since, in the case of linear interpolation within a triangulation-based DTM program, only three points are used for interpolation, a value of 3 for C in eq. 1 is assumed to be the approximate value, based on analogy with the law of error propagation, although one would never obtain the exact value because the shape of triangles varies greatly. The K-values can then be computed from the results shown in Table 4 and they range from 4.5 to 5.9. These results show that the error budget coming from the loss in fidelity of terrain topography which is selectively represented by only contours is in the range of $CI/4$ to $CI/6$, depending on the characteristics of terrain topography.

It is interesting to note that after the F-S data have been included, the values of σ have been reduced to a level of $CI/6$ to $CI/15$ from their original level of $CI/3$ to $CI/5$. The improvement in the values for these three areas ranges from about 40% to 60%. These results are also in accordance with the results obtained by Tuladhar and Makarovic (1988), where an improvement of 53% in the RMSE value was reported. Also the magnitude of the residual errors can be significantly reduced. The corresponding accuracy results are also given in Table 4.

If the values of the DTMs obtained from the contour data plus the F-S data are also expressed in terms of "per mil of H", then the values are from about 0.2 to 0.25. Further analysis using eq. 1 can also be carried out again; the results for K range from about 9 to 28.

These results show that, after the F-S data are added into the contour data, the fidelity of the terrain topography is greatly improved. Therefore, the error component coming from the loss in fidelity of the terrain topography, which is selectively represented by contour data and F-S data, can be expected to be within a range of $CI/30$ to $CI/10$, depending on the characteristics of terrain topography.

5. Improvement in the accuracy of DTMs derived from square-grids with additional feature-specific data

In the previous section, accuracy results for the DTMs derived from contour data have been reported. This section describes the accuracy results for the DTMs derived from square-grid data both with and without the additional F-S data.

The square-gridded data sets have also been tested against the check points using the procedure described previously. For these grids, a new approach has been developed by the author to generate a series of new grids with various (but larger) intervals from each of the original grids. The test results from all these grids have been reported in another paper (Li, 1992). Here, only some results (i.e. σ-values) which are related to this study are quoted in Table 5.

In the same table (Table 5), the accuracy results for the DTMs derived from the square-gridded data with additional F-S data are also quoted.

TABLE 5

DTM accuracy (σ-value) improvement by adding feature-specific points to the grid data

Test area	Grid interval (m)	Standard error (σ; m)		Difference in σ-values (m)	Ratio in grid interval
		with F-S	no F-S		
Uppland	28.28	0.63	0.59	0.04	1.000
	40	0.76	0.66	0.10	1.414
	56.56	0.93	0.70	0.23	2.000
	80	1.18	0.80	0.38	2.828
Sohnstetten	20	0.56	0.40	0.16	1.000
	28.28	0.87	0.55	0.32	1.414
	40	1.44	0.77	0.67	2.000
	56.56	2.40	1.08	1.32	2.828
Spitze	10	0.21	0.14	0.07	1.000
	14.14	0.28	0.15	0.13	1.414
	20	0.36	0.16	0.20	2.828

F-S denotes the feature-specific data.

From this table, it can be seen that, with additional F-S data, the accuracy of DTMs is much improved. The results seem to confirm that the accuracy of DTMs has a linear relationship with grid interval noted by Ackermann (1979), but only when F-S data are included. If F-S data are not included, the relationship is parabolic. The relationship is very clear for the results obtained from the Uppland and the Sohnstetten areas and these results are plotted in Fig. 2.

It is of interest to compare these σ-values in Table 5 to see if there is any inherent relationship between the accuracy of a DTM derived from square-grid only and that with additional F-S data. To do so, it is desirable first of all to carry out some regression analysis to see if there is a clear relationship between these values. Indeed, such analysis has been carried out elsewhere (Li, 1990, 1992). Here a discussion will be conducted from another point of view.

The model tested was that the difference in σ-values is proportional to the square of d (grid interval), which can be expressed mathematically as follows:

$$\Delta\sigma = \sigma_g - \sigma_c = A + B \times d^2 \quad (2)$$

where d is the grid interval; A and B are two constants; σ_c and σ_g are the σ-values for the DTMs derived from grid data with and without F-S data, respectively; and $\Delta\sigma$ represents the difference in the two σ-values.

This model is not arbitrarily selected. It is based on the mathematical models of accuracy of DTMs, which were developed by the author (Li, 1994) through theoretical analysis of DTMs based on

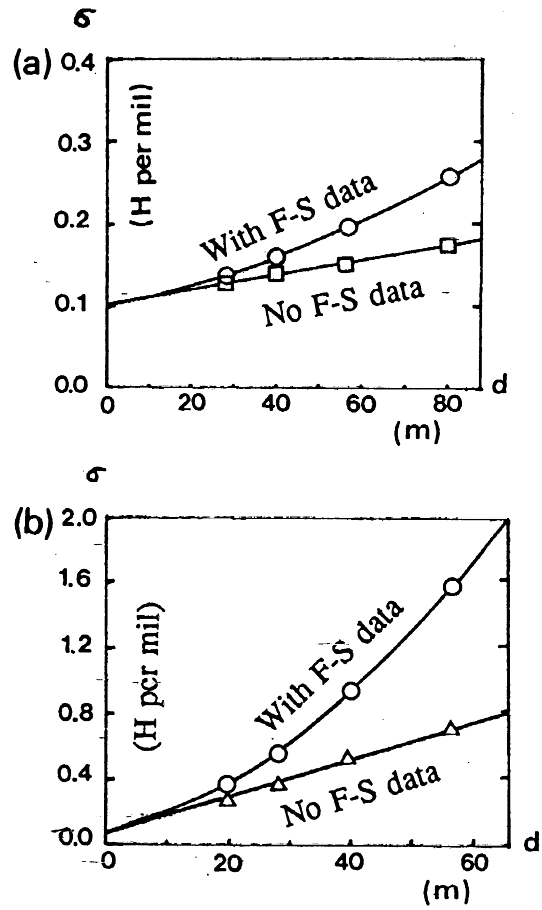


Figure 2. Variation of DTM σ-values with grid intervals. (a) For the Uppland area: upper without F-S data, and lower with F-S data. (b) For the Sohnstetten area: upper without F-S data and lower with F-S data.

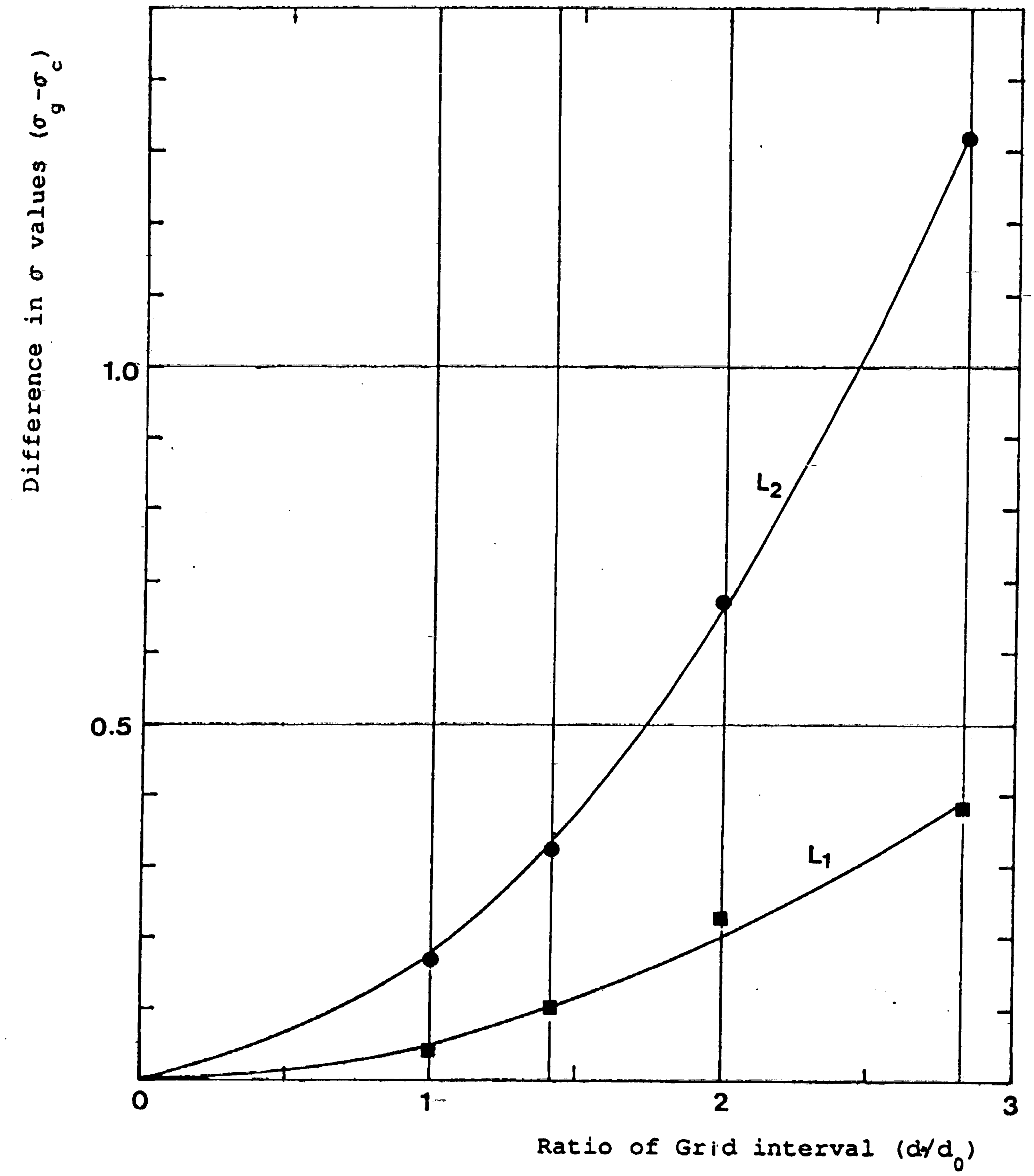


Figure 3. The relationship between the difference in value (between the DTMs derived from grid data only and that with F-S data) and the ratio of grid interval. Points (either circles and squares) represent tested results, with continuous lines for regression results. L_1 and L_2 are for Uppland and Sohnstetten areas, respectively. They are quadratic curves, where $D = d/d_0$ is the ratio of grid interval d divided by the smallest grid interval d_0.

both square grid only and square grid plus F-S data.

These $\Delta\sigma$-values (in Table 5) are plotted in Fig. 3 with corresponding curve lines which were obtained by applying regression analysis to these values using eq. 2. (The results for the Spitze area are not used since there are only three data points.) The values of the constant A in the equations of these two curve lines (L_1 and L_2 in Fig. 3) are very close to zero. Therefore, the following expression seems to apply in this case:

$$\frac{\Delta\sigma_2}{\Delta\sigma_1} = \left(\frac{d_2}{d_1}\right)^2 \tag{3}$$

where d is the grid interval, $\Delta\sigma_1$ and $\Delta\sigma_2$ represent the difference in values corresponding to d_1 and d_2.

6. Comparison of the accuracy of contour-based DTMs with grid-based DTMs

In the previous two sections, some discussion of the accuracy of the DTMs derived both from contour data and grid data has been presented. A comparison will be made in this section.

To compare the quality of two DTMs, it is helpful to have a comparison of data density between them. In the case of DTMs derived from contour data, the data density is expressed in terms of contour interval (CI), while in the case of DTMs derived from gridded data, the data density is expressed in terms of grid interval (d). To do such a comparison, the average value of the planimetric contour interval, denoted as D here, should be used. Here,

$$D = CI \times \cot\alpha \qquad (4)$$

where α is the mean slope angle of the terrain surface, CI is the contour interval and D is the average planimetric contour interval. Taking the test data in this study as an example, the average planimetric contour intervals corresponding to these three areas are approximately as follows: 50 m for Uppland, 20 m for Sohnstetten and 10 m for Spitze.

Theoretically, the accuracy of the DTMs derived from contour data with an interval CI should be approximately equal to that derived from the grid data with grid interval of D as expressed in eq. 4, if the terrain surface is homogeneous; the effect of the errors involved in source data on the final results are the same.

At this stage, it is of interest to examine the practical results, which are shown in Fig. 4. It shows that, for the Uppland area, the accuracy of the DTM data, in terms of σ, derived from the contour data with a 5 m vertical interval, is much lower than that derived from a 50 m grid. It cannot match that derived from the large grid with an interval of 80 m. For the Spitze area, the accuracy of the contour data (also in terms of σ) with 1 m interval is also lower (i.e. σ-value larger) than that derived from a 10 m grid, but is higher than that derived from grid data with an interval of 14 m. However, with the same contour interval as that for Uppland, the contour data for Sohnstetten can match the grid data with a grid interval of about 30 m, but still not 20 m.

Even if the fact that the accuracy of contour data (in terms of σ) is lower than that of grid data is excluded according to eq. 1, the accuracy of the DTMs derived from contour data is still lower than that derived from the grid data with a grid interval equal to the average planimetric contour interval computed from eq. 4. From these limited results, it seems that the value of matchable or equivalent grid interval (denoted as d here) is 1.5 to 2.0 times larger than the D-value computed from eq. 4. This may be due to the fact that the grid data are more evenly distributed than contour data. Also

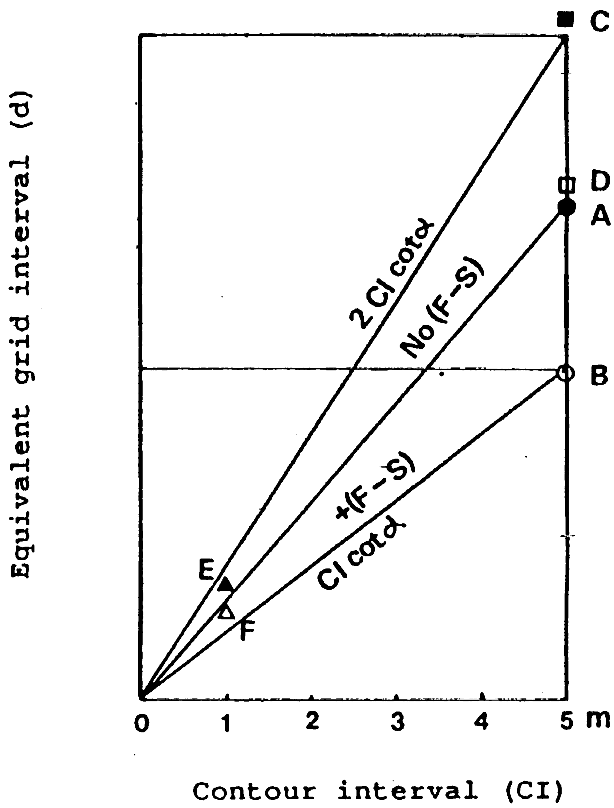


Figure 4. Practical guide for the relationship between contour interval *(CI)* and its equivalent grid interval (*d*). Points *B* and *A* represent the test results for the Uppland area with and without F-S points, respectively. Points *D* and *C* represent the test results for the Sohnstetten area with and without F-S points, respectively. Points *F* and *E* represent the test results for the Spitze area with and without F-S points, respectively.

this phenomenon might be due to the result of imprecise estimates of slope angles. Figure 4 also shows that after the inclusion of F-S data, the values of K can be reduced to a level of about 1.0 to 1.5. Therefore,

$$d = K \times D = K \times CI \times \cot\alpha \qquad (5)$$

where K is a constant and ranges from 1.5 to 2.0 in the case without the inclusion of F-S data and from 1.0 to 1.5 in the case with F-S data.

One particularly interesting point is that, for the Sohnstetten area, the accuracy of the DTM (in terms of σ) derived from contour data with additional F-S data is better than that derived from grid data with 20 m interval with additional F-S data (i.e. K <1.0). This might be due to the fact that the contour data, after the addition of F-S data, are much more evenly distributed than before and that more important points are missed in the case of contour data than in the case of gridded data. This might suggest that, in areas with relatively steep slope and smooth terrain such as the Sohnstetten area, contouring could be a better sampling strategy than regular-grid sampling.

7. Concluding remarks

From this study, the following remarks are made:

(1) The accuracy of the DTMs (in terms of RMSE or σ) derived from photogrammetrically measured contour data only is about $CI/3$ to $CI/5$, depending on the characteristics of the terrain topography. With additional F-S data, the accuracy of the resulting DTMs is greatly improved and the value ranges from about $CI/6$ to $CI/14$, depending on the characteristics of the terrain topography. For the sake of safety, the value of $CI/6$ could be used in practice. The improvement in DTM accuracy is from 40 to 60%; a value of 40 to 50% can be considered in practice;

(2) The relationship between the accuracy of DTMs (in terms of σ) derived from gridded data plus data and grid interval is linear. But, if F-S data are not included, the relationship becomes parabolic. The amount of difference between the σ-values obtained in these two different cases (i.e. with and without F-S data) becomes greater with an increase in grid interval and it seems that the rate for the former is the square of the rate for the latter.

(3) When comparing the accuracy of the DTM derived from grid data with that derived from contour data, it can be found that contour data with an interval of CI can only match the grid data with grid interval of approximately ($K \times CI \times \cot\alpha$), where K ranges from 1.5 to 2.0 if F-S data are not included. However, if additional F-S data are used, the K-value in eq. 5 then ranges from 1.0 to 1.5.

(4) From the results obtained in this study, it is suggested that contouring could be a better sampling strategy than regular-grid sampling in areas with relatively steep slopes and smooth terrain such as the Sohnstetten area.

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