This paper describes some algebraic models for the automated generalization of area-patches. These models are based on techniques developed in mathematical morphology, which is a science dealing with the form and shapes of objects, and will form an algebraic basis for area-patch generalization. These models are fully illustrated and tested. The results show that such models are very promising.

An algebraic basis for digital generalization of area-patches based on morphological techniques

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INTRODUCTION

Generalization is a traditional topic in cartography and it has become increasingly important in the digital era with the widespread use of GIS. Indeed, generalization is so important and difficult a topic that it has nowadays become a major international research theme in cartography and geographic information sciences (Marble, 1984, Abler, 1987; Rhind, 1988; Müller, 1991). Over the last decade, many projects have been initiated internationally, in Canada, China, Britain, France, Germany, the Netherlands, Sweden, Switzerland, and the USA. Indeed, generalization of cartographic data and other spatial information has become one of only twelve initiatives of the NCGIA, under the title of "Multiple Representations". Also, the European GISDATA Task Force, which can be considered as a European analogy to NCGIA, has identified generalization as a key issue. In the cartographic community, the International Cartographic Association (ICA) has recently established a working group on this subject.

In spite of international efforts for so many years, not much real progress has been made. So far, most research efforts in the area of generalization have been spent on line simplification in vector mode. Generalization research in the raster domain has been lacking at the international level and only a very few researchers (e.g. Monmonier, 1983) have considered raster data. However, it should be more convenient to carry out generalization operations in raster mode, since generalization is caused by reduction in space when scale is reduced, and raster is a space-primary data structure. This paper will present some algebraic models for the generalization of area-patches in raster mode, based on techniques developed in mathematical morphology.

The introduction will be followed by a brief discussion of the problem of generalizing area-patches in a digital environment and a review of previous studies. Then the techniques developed in mathematical morphology will be introduced. After that, algebraic models for area-patch generalization will be presented complete with illustrations.

THE PROBLEM AND PREVIOUS STUDIES

There are some situations where small area features with the same semantic meaning spread over a certain area. *Figure 1* shows an example, which is adopted from Müller and Wang (1992). Such is a case of area-patches. These small area features could be forest coverage, land use, islands or water bodies, mining deposits, etc. These area features could vary in size, shape and spatial distribution (Müller and Wang, 1992).

When map scale is reduced, some areas features will be too small to be shown, some will touch and some will have coalesced. Therefore, some features need to be eliminated, some displaced, and some combined. In other words, a generalization process including feature elimination, displacement, and combination should be applied. The problem arising is how to apply this generalization process. The following are some general rules (see Müller and Wang, 1992):

(a) Only a subset of the original patches is preserved after scale reduction and the remaining patches should be exaggerated.
(b) Elements of the counter of the convex hull delimiting the original distribution of patches must be recognizable after generalization; and

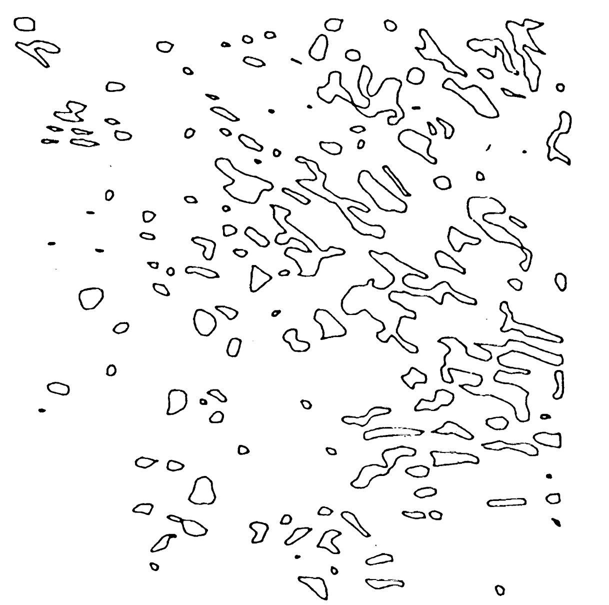

Figure 1. An example of area-patches. (after Müller and Wang, 1992).

(c) The structure of the spatial distribution of patches must be preserved.

Considering these rules, Müller and Wang (1992) have developed a procedure for area-patch generalization as follows:

(a) Rank all patches by size and calculate the cumulative areas;
(b) Determine the patches to be expanded and contracted;
(c) Perform expansion or contraction;
(d) Perform elimination (for area feature that are too small);
(e) Reselect a few eliminated area features in regions where they are sparsely distributed;
(f) Expand all area features;
(g) Merge overlapping or area features that touch;
(h) Displace coalesced features;
(i) Verify topological integrity; and
(j) Smooth the contour of patches.

This is a procedure for vector data. It must be difficult to perform expansion, contraction, displacement and merging in vector format. Here a new solution is provided for area-patch generalization in raster mode. If the original data is in vector format, then one needs to (1) rasterise the data, (2) perform the generalization process and then (3) vectorise the generalized results.

MORPHOLOGICAL TECHNIQUES FOR ALGEBRAIC BASIS

The algebraic models to be presented in this paper are based on techniques developed in mathematical morphology, as it has been revealed by a recent study carried out by Li (1994) that these techniques are very useful tools for digital generalization.

Mathematical morphology is a science of form and structure, based on set theory. It was developed by French geostatistical scientists Matheron and Serra in the 1960s (Matheron, 1965; Serra, 1982). Since then it has found increasing application in digital image processing. Efforts have also been made by researchers for applying morphological tools to mapping related sciences, such as in digital terrain modelling (Li and Chen, 1991).

The basic morphological operators are dilation and erosion. They are defined as follows (see Serra, 1982; Haralick *et al.*, 1987):

$$\text{Dilation:}\quad A \oplus B = \{a + b: a \in A, b \in B\} = \cup_{b \in B} A \qquad (1)$$

$$\text{Erosion:}\quad A \ominus B = \{a : a + b \in A, b \in B\} = \cap_{b \in B} A_b \qquad (2)$$

where A is the image to be processed and B is called the structuring element, which can be considered to be an analogy to the kernel in convolution operations. In Eq. (1), it is called "dilation of A by B" and in Eq. (2) "erosion of A by B". Examples of these two operators are given in *Figure 2*, where the features are represented by pixels of "1"s and the origin of the structuring element is marked with a circle.

The structuring element is a critical element in these operations. It could take any shape, e.g. a 2 × 2 or 3 × 3 image. *Figure 3* shows some of the possible shapes. If a symmetrical structuring element with origin at the centre is used for dilation, then the shape of the original image will be expanded uniformly along all directions; thus the dilation in this particular case is called expansion. Similarly, the erosion in this case is called shrink. These two special operations are illustrated in *Figure 4*(c) and (d).

Based on these two simple operators, i.e. dilation and erosion, a number of new operators have also been developed, such as closing, opening, thinning, thickening, sequential dilation, and conditional sequential dilation, and so on. However, it is not the purpose of this paper to discuss all of them. More detailed information can be found from the book by Serra (1982).

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Figure 2. Two basic morphological operators, dilation and erosion: (a) Original image A, (b) The structuring element B, (c) A dilated by B ($A \oplus B$), (d) A eroded by B ($A \ominus B$).

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                             1 1 1
        (a)                   (b)

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S3:   1 1 ① 1 1       S4:    1 ① 1
      1 1 1 1 1                 1
      1 1 1 1 1
        (c)                   (d)

                               1
          1           S6:       ①
S5:     1 ① 1                     1
         (e)                   (f)
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Figure 3. Some examples of possible structuring elements: (a) A 2 × 2 squared pattern, (b) A 3 × 3 squared pattern-"H" pattern, (c) A 5 × 5 squared pattern, (d) A "Cross" type pattern. (e) A triangular pattern. (f) A diagnonal pattern.

ALGEBRAIC MODELS FOR AREA-PATCH GENERALIZATION

The procedure used by Müller and Wang (1992) is for the process of vector data. However, for this process in raster mode, it needs to be modified. The procedure developed in this study is as follows:

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Figure 4. Special cases of dilation and erosion, expansion and shrinking: (a) Original image A, (b) The structuring element H, (c) A expanded by H ($A \oplus H$), (d) A shrunken by H ($A \Theta H$).

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(d)

Figure 5. Elimination of small area-patches by erosion: (a) The original image P, as a portion of area patches, (b) The image P eroded by S1 (E10 = $P \Theta$ S1), (c) The image P eroded by S2 (E20 = $P \Theta$ S2), (d) The image P eroded by S3 (E30 = $P \Theta$ S3).

(a) Determine the size of structuring element;
(b) Apply erosion to all area features (to eliminate too small areas);
(c) Restore those survived after erosion;
(d) Apply dilation to all the area features after restoration;
(e) Apply erosion to the dilated data (image), if desirable; and
(f) Apply the post-processing, if desirable.

The determination of the size of structuring elements will be based on the scale of source data and target as well as the pixel size of the raster data. This problem will be discussed later.

The first step in the algebraic process of area-patch generalization is erosion:

$$\mathrm{E} = P \,\Theta\, \mathrm{B}_e \qquad (3)$$

where, P is the original image of area patches; B_e is the structuring element; and E is the result after erosion.

By applying an erosion process to the original data (image), all area features smaller than the structuring element will be eliminated. *Figure 5* shows the case (a) is the original image P, being a portion of an large area. (b) is the image P eroded by structuring elements S1 (i.e. B_e = S1), (c) is the result of P eroded by S2 (i.e. B_e = S2) and (d) is the result of P eroded by S3 (i.e. B_e = S3). It can be seen that some area features are too small to survive during the erosion process. In other words, some small area features are eliminated. For vector data, if a feature is to be eliminated, one can simply delete it from the data file. However, in raster format, the story is different.

After erosion the size of every area feature is reduced. Some may have been completely eroded; some may have only one pixel left; while other have more pixels left. The next step is called restoration, i.e. to restore the area features which have not been completely eroded. To achieve this goal, the following algebraic formulae can be used:

$$R_k = (R_{k-1} \oplus B_r) \cap P \qquad (4)$$

Where, R_k is the result of Kth round of restoration; B_r is the structuring element, P is the original image before erosion and

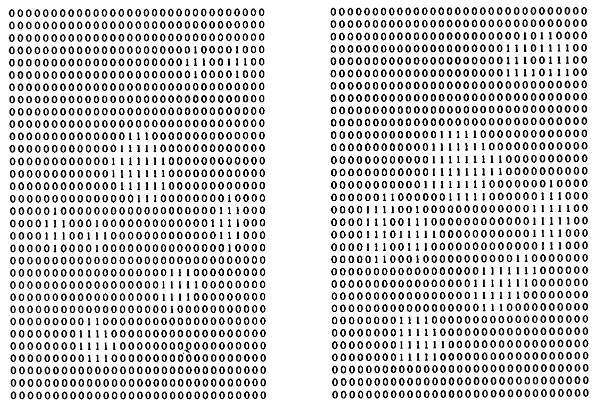
$$R_0 = E \qquad (5)$$

The restoration process will stop if the following condition holds:

$$R_k = R_{k-1} \qquad (6)$$

The restoration process is illustrated by *Figure 6*, where structuring element S4 as B_r in Eq. (4) and E20 as E in Eq. (5) are used. A discussion of why this structuring element is used will be conducted later. *Figure 6*(a) is the result after first round of restoration and *Figure 6*(b) is the final result after four rounds of processing.

The restored image represents the area patches after elimination. The next step is to apply a dilation process to this image. This process serves three purposes. Firstly, area features will be exaggerated. Secondly, the area features with small spacing will be combined to form bigger areas. Thirdly, the contours of the patches will be smoothed. *Figure 7* shows the dilated result. This process contains the contents of steps (f), (g) and (j) in the



(a) (b)

Figure 6. Restoration of area patches survived after erosion: (a) Result of first round restoration by S4 (R21 = (E20 ⊕ S4) ∩ *P*) (b) Final result after four rounds of restoration processing (R24 = (R23 ⊕ S4) ∩ *P*).

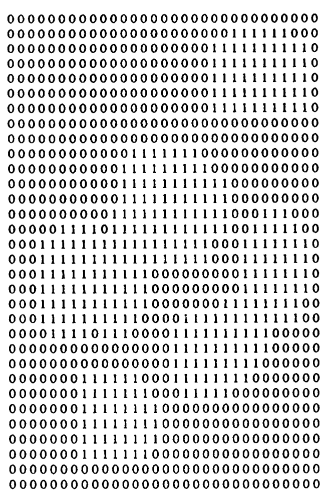


Figure 7. Dilation of restored image R24 (D = R24 ⊕ S2).

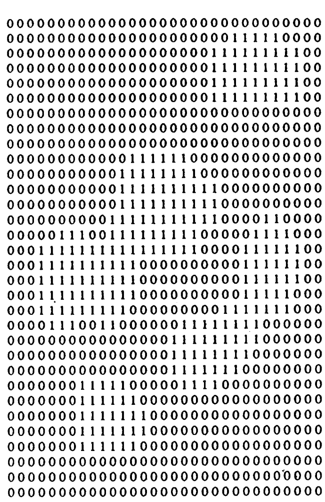


Figure 8. Erosion of Fig. 7 by S1 (F = D ⊖ S1).

procedure developed by Müller and Wang (1992). The algebraic expression of this dilation process is as follows:

$$D = R_k \oplus B_d \quad (7)$$

where, R_k is the restored image, B_d is the structuring element and D is the dilated image.

It seems desirable to apply an erosion process to the dilated image, but using a structuring element smaller than the one used in Equation (7). In this way, exaggeration of area features can still be retained somehow while the need for the displacement of coalesced features, which is the step (h) in the procedure developed by Müller and Wang (1992), is avoided. As a consequence, also there will be no need for verifying the topological integrity. *Figure 8* illustrated the results, image *F*, after applying an erosion process to the image *D*.

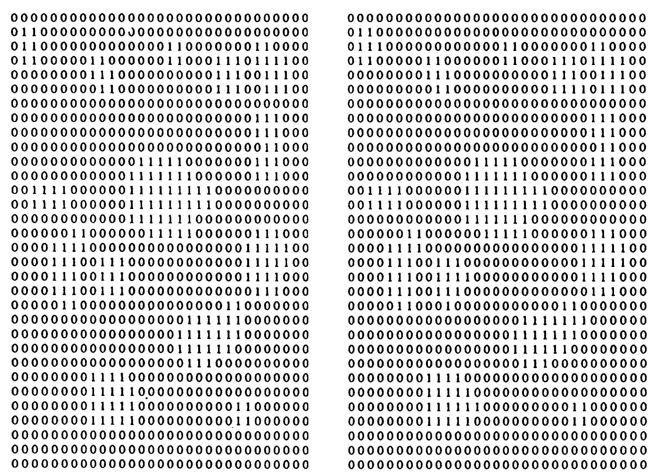
DISCUSSION

The selection of structuring elements is a critical operation. The size of structuring elements should be determined according to the scales of source and target map and the size of raster pixels. In general, a size from 0.5 to 0.7 mm at target map is appropriate, based on the authors' experience.

It is also desirable to use structuring elements with a symmetrical shape for the restoration process. *Figure 9* shows the problem when an asymmetric structuring element is used in the restoration process: that is, some pixels are not restored.

It is advisable to use small structuring elements in the restoration process. The problem of using oversized structuring elements is illustrated in *Figure 10*. In this figure, a 5 × 5 structuring element is used. In the second round of restoration, the dilated image is far too big already so that it covers the areas of other eliminated features. When this dilated image intersects the original patch *P*, one pixel in the other (eliminated) area is also restored as shown in *Figure 10*(b). This effect continues and the final result is that all small areas at the lower/left corner are completely restored as shown in *Figure 10*(d), although they were all eliminated. For the sake of cautions, the authors would suggest structuring element, S4 in *Figure 3*, be the candidate.

In those portions of the area where features are sparsely distributed, if all small features are eliminated, then the structure of the area patches will be altered. As suggested by Müller and



(a) (b)

Figure 9. The problem of using asymmetric structuring element for restoration: (a) First round of restoration by S1 (R11 = (E10 ⊕ S1) ∩ *P*) (b) Final result of restoration: many areas are not restored. (R14 = (R13 ⊕ S1) ∩ *P*).

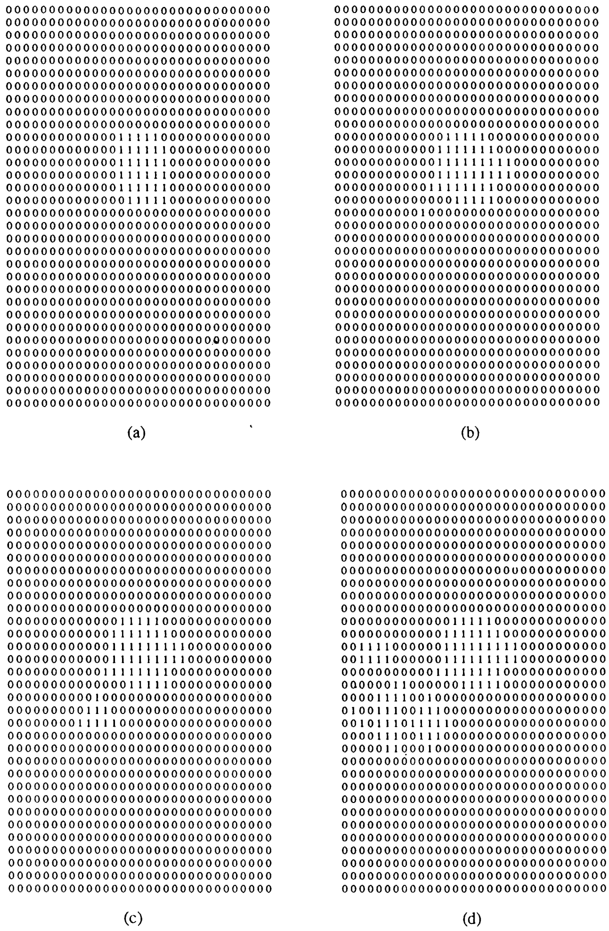


Figure 10. The problem using oversized structuring element: (a) Result of first round restoration by S3 (R31 = (R30 ⊕ S3) ∩ P), (b) Result of third round restoration by S3, a pixel outside desirable area is restored (R33 = (R32 ⊕ S3) ∩ P), (c) Result of fourth round restoration by S3 (R34 = (R33 ⊕ S3) ∩ P), (d) The final result after eigth round restoration by S3: many undesirable small areas are restored (R38 = (R37 ⊕ S3) ∩ P).

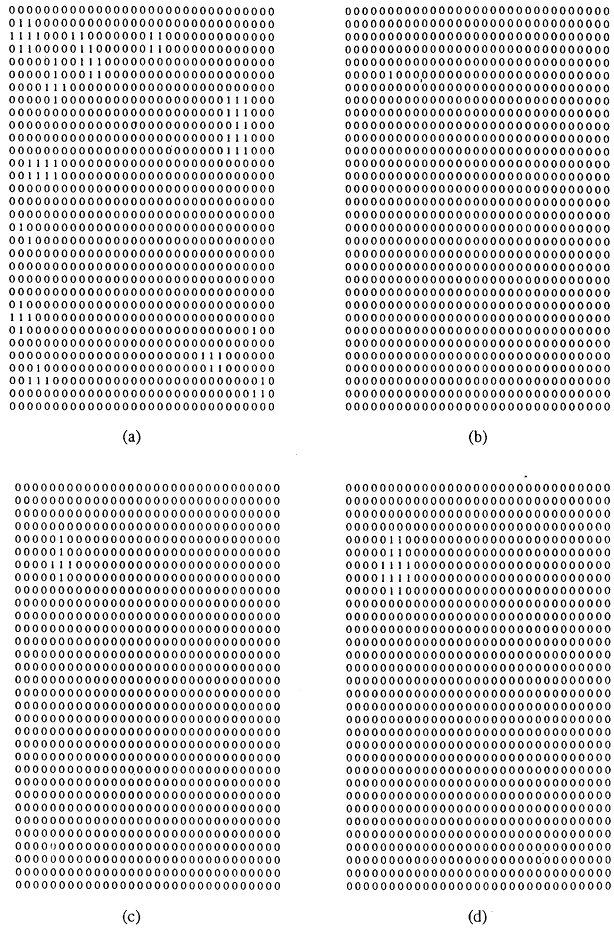


Figure 11. Addition of a "lucky loser" in a sparsely distributed region: (a) Smaller areas eliminated (Q = P − R24), (b) Image L: A pixel is selected which is located in the area of a lucky loser, (c) The lucky loser is restored RL12 = (RL11 ⊕ S4) ∩ P) (RL11 = (L ⊕ S4) ∩ P), (d) The restored lucky loser is dilated to meet the minimum size required (DL12 = (RL12 ⊕ S1)).

Wang (1992), a few of them need to be reselected although they are too small in size. By a close examination of the restored image R, it can be found that the small features at the upper/left corner are all eliminated during the erosion process. In this case, one small feature is to be reselected and this one is called the lucky loser here.

The selection of the 'lucky loser' is very simple, however. One simply creates an image L with the same size as P, with all pixels being O's. By examining the difference between the restored image and the original image as shown in *Figure 11*(a), one then changes one pixel of the image L into 1 in the area occupied by the 'lucky loser', as shown in *Figure 11*(b). Then, a similar restoration process is applied. The restored image, as shown in *Figure 11*(c), will be too small to be represented. Then, a dilation process is applied to the restored 'lucky loser,' as shown in *Figure 11*(d). The dilated 'lucky loser' can then be added to image F to obtain final image, as shown in *Figure 12*. The addition of 'lucky losers' can be considered as being a post-processing.

CONCLUDING REMARKS

This paper describes an algebraic basis for the generalization of area patches. As can be seen, these techniques are extremely flexible. Indeed, they can handle the problem of generalizing area-patches very well. The techniques used in this paper are in raster mode. If the original data is in vector format, then one needs to:

(a) rasterise the vector data;
(b) perform displacement operation; and
(c) vectorise the generalized results.

There are many advantages of this technique over other techniques as follows:

(a) it is space-primary, no further spatial conflicts will be created;
(b) it is based on simple set operations, so it is very fast in terms of computation;
(c) it provides an algebraic basis, so it makes such a process more scientific and more objective; and
(d) it largely automates this process.

This is the result of a preliminary study. Further research into how the shape and size of structuring elements affect the generalization process is being carried out by the authors. Special consideration is given to the use of the *natural principle* (Li and Openshaw, 1993) to guide the process.

Indeed, morphological techniques, as illustrated by Li (1994), are good not only at generalization of area-patches but also at other operations in generalization such as feature displacement (Li and Su, 1995), smoothing, etc., Further research in these areas is being carried out by the authors.

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(N=F + DL12)

Figure 12. Final result of generalisation after addition of the lucky loser.

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REFERENCES

Abler, R., 1987. The National Science Foundation National Center for Geographic Information and Analysis. *International Journal of Geographical Information Systems*, 1(4): 303–326.

Haralick, R., Sternberg, S. and Zhuang, X., 1987. Image analysis using mathematical morphology. *IEEE Transactions of Pattern Analysis and Machine Intelligence*, 9(4): 532–550.

Li, D. and Chen, X-Y., 1991. Automated generating triangualted irregular digital terrain model networks by mathematical morphology. *ISPRS Journal of Photogrammetry and Remote Sensing*, 46: 283–295

Li, Z., 1994. Mathematical Morphology in digital generalisation of raster map data.*Cartography*, 23(1): 1–10.

Li, Z. and Openshaw, S., 1993. A natural principle for the objective generalisation of digital maps. *Cartography and Geographic Information Systems*, 20(1): 19–29.

Li, Z., and Su, B., 1995. Feature displacement in the generalization of digital map data using morphological techniques. Submitted to *Cartographica*.

Marble, D., 1984. Geographical information systems: An overview. *Proceedings of Pecora 9*, Sioux Falls, S. D. 18–24. Reprinted in *Introductory Readings in Geographic Informations Systems*, edited by D. J. Peuquet and D. F. Marble. Taylor and Francis, London. 8–17.

Matheron, G., 1965. *Random Sets and Integral Geometry.* Wiley, New York.

Monmonier, M., 1983. Raster-mode area generalisation for land use and land cover maps. *Cartographica*, 20(4): 65–91.

Müller, J.-C., 1991. The cartographic agenda of the 1990s: updates and prospects. *ITC Journal*, 1992–2: 55–62.

Müller, J.-C. and Wang, Z.-S., 1992. Area-patch generalisation: a competitive approach. *The Cartographic Journal,* 29(2): 137–144.

Müller, J.-C.., Weibel, R., Langrange, J. P and Salge, F., 1993. Generalization: State of the art and issues. *GISDATA Task Force on Generalization 1993.* European Science Foundation. 21 pp. Also in: *GIS and Generalization,* edited by J.-C Müller, J. P. Lagrange and R. Weibel, Taylor and Francis. 1995. 3–17.

Rhind, D., 1988. A GIS research agenda. *International Journal of Geographical Information Systems*, 2(1): 23–28.

Serra, J., 1982. *Image Processing and Mathematical Morphology.* Academic Press, New York. 610pp.