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Morphological Models for the Collapse of Area Features in Digital Map Generalization

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Abstract

"Collapse" is an essential operation for the manipulation of area features in digital data generalization. This operation can be categorized into two types: complete collapse and partial collapse. The former is composed of another two types: area-to-point and area-to-line collapse. In this paper, a set of algebraic models built upon the operators in mathematical morphology is described for the area-to-line collapse and partial collapse operations. For the area-to-line collapse operation, a modified skeleton algorithm is presented. For the partial collapse operation, a procedure is designed which consists of a set of operations, i.e., the skeletonization, separation of areal and linear parts, simplification of areas and an overlay operation. These models are tested using real map data sets.

Keywords: map generalization, collapse operation, partial collapse, mathematical morphology

1. Introduction

Map generalization is one of the issues related to scale, i.e., multi-scale representation. These scale issues are fundamental to all spatial sciences but unfortunately they still remain unsolved. Furthermore, it is believed by researchers [6], [11] that little has been understood regarding scale problems. Of course, one may argue that map generalization must already have been understood, because this is an old cartographic problem that has attracted much attention from both the GIS and cartographic communities in this digital era. Indeed, many projects on this topic have recently been initiated by researchers from all over the world, including Australia, China, Germany, France, Hong Kong, the Netherlands, Sweden, Switzerland, the UK, USA and so on. However, progress is still very slow. This is evidenced by the fact that no really successful generalization systems have so far been marketed.

Through an examination of the literature, it is clear that most research in digital map generalization has been on the generalization of line features. However, the most popular algorithm produces inferior results, such as self-interaction [28]. Indeed, most such algorithms have similar problems except a few, e.g., those by Li and Openshaw [12], Wang and Müller [26] and de Berg et al. [3] (see [28]). More recently, attention has been given to the rule-based approach. However, it has been noticed that only some general rules can be formalized, which are not concrete (precise) enough for system implementation. If one goes down to the bottom level, one discovers that most generalization operations are still at a conceptual level. In other words, for most of these operations, no algorithms or mathematical models have been developed. And the "partial collapse" operation (see Section 2) seems to be one of these, although some work on the (complete) area-to-line collapse has already been done [2], [4], [9]. Therefore, the main objective of this paper is to present a mathematically elegant solution to the partial collapse operation. Such models might be compared to other transformation models, such as affine and conformal models. This represents part of an overall aim to develop a "toolbox" for the full range of relevant operations required for generalization in a digital environment.

In terms of relevant methodology, it is notable that most research into automated map generalization is currently focused on vector data, even for area features, with some exceptions, e.g., Monmonior's work in the 1980s [16], the doctoral theses at the beginning of the 1990s by Jäger [8] and Schyberg [18] as well as the more recent work by Li and Su [10], [13], [21], [22]. However, it should be more convenient to manipulate area features in raster mode since raster is a space-primary data structure. This study will concentrate on raster data, in other words, the algebraic models developed in this study are in raster mode. More precisely, they are built upon the operators developed in mathematical morphology, which is a science of shape, form and structure.

This introduction is followed by a discussion of the collapse problem in general (Section 2). Then the mathematical background of mathematical morphology (MM) is introduced (Section 3). Based on the MM operators, a skeletonization algorithm is modified and presented (Section 4). This can be used for both the area-to-line (complete) collapse operation and as part of the partial collapse procedure (Section 5). The morphological models developed in this study are experimented with using a set of real map data with a discussion of the results (Section 6).

2. Collapse operation: the problem and possible solutions

Collapse is an operation to transform area features to point features, or to transform area features completely/partially to linear features, to suit the representation at a smaller scale. Accordingly, a collapse operation can be categorized into two types: complete collapse and partial collapse. Complete collapse is composed of another two types: area-to-point collapse and area-to-line collapse. A simple illustration of such collapse operations is shown in figure 1.

The area-to-point collapse operation transforms an area feature into a point feature when it appears to be too small to be represented as an area feature at a smaller scale. Normally, resulting point features are positioned at the center of gravity of the original area feature. The computation of such a position is not a difficult task and this operation will not be further discussed here.

The area-to-line collapse happens when an area is a long but thin feature. In this case,

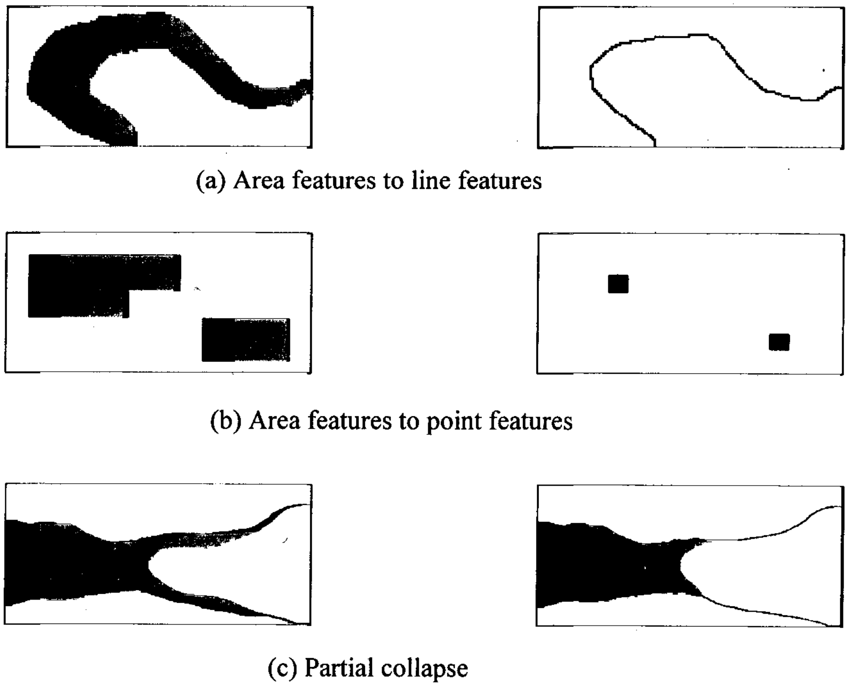


Figure 1. Simple examples for collapse operations.

the feature will appear to be too thin to be represented as an area feature at a smaller scale. As a result, such features will be represented as a line feature at a smaller scale. The main task for this operation is to derive an appropriate line to represent this original feature. Among many candidates, the skeleton of the original area feature seems to be a logical choice. In fact, this candidate has been widely accepted and derivation of appropriate skeletons has been considered by several researchers [2], [4], [9]. However, most of the previous work discusses the use of triangular data structures to support the derivation of skeletons in vector mode. This paper will discuss an algorithm in raster mode.

Partial collapse occurs when a part or some parts of an area feature are long but thin. The main tasks of this operation are (a) to make these long but thin parts a complete collapse and (b) then to simplify (or smooth the boundary of) the part of the area feature which is not collapsed. This is a more complicated operation and the authors are not aware of any literature on its realization. This operation is the main topic of this paper.

To achieve an appropriate result for the partial collapse, the critical issue to be addressed is how to integrate the two steps (a) and (b) described in the previous paragraph. In fact, a number of other supporting operations will also be required in this integration process and a more detailed discussion on an integrated procedure based on the operators developed in mathematical morphology will be conducted later. However, before this procedure can be described, a brief introduction to mathematical morphology seems necessary and will be given next.

3. Background: mathematical morphology

In this paper, all the techniques to be discussed are built upon the operators of mathematical morphology (MM), which was formulated by two French geostatistical scientists—G. Matheron and J. Serra in the early 1960s [14], [19]. MM has found wide application in image processing and other areas. It has also been applied to digital map generalization [10], [13], [21], [22], [23].

3.1. Basic morphological operators

For the convenience of discussion, the two basic operators are defined here as follows (see [7], [19]):

Dilation: $$A \oplus B = \{a + b : a \in A, b \in B\} = \cup_{b \in B} A_b \quad (1a)$$

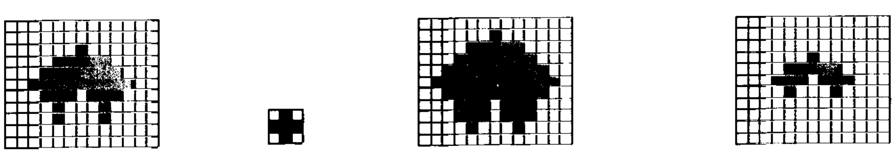
Erosion : $$A \ominus B = \{a : a + b \in A, b \in B\} = \cap_{b \in B} A_b \quad (1b)$$

where $\ominus$ and $\oplus$ stand for erosion and dilation operations, A is the image to be processed and B is called the structuring element, which can be considered to be an analogy to the kernel in convolution operations. In Equation (1a), it is called ''dilation of A by B'' and in Equation (1b) ''erosion of A by B''. It should be noted that the symbol '' $+$ '' represents vector addition. Thus, ''a'' is translated by the coordinates of ''b''. The symbol A_b represents the translation of all elements of A by ''b''.

Examples of these two operators are given in figure 2, where the features are represented by black pixels. The origin of a structuring element is at its geometric center if there is no other specific indication.

The structuring element is a critical element in any morphological operation. It can take any shape (square, cross) and size (e.g., 2×2 or 3×3). Figure 3 shows some of the possible shapes, i.e., circular, diagonal, linear, square, triangle, etc.

Here, ''circular'' means that it is used to approximate a circle in mathematical morphology.



(a) Original image A (b) Structuring element B (c) A dilated by B (A⊕B) (d) A eroded by B (A⊖B)

Figure 2. Two basic morphological operators: dilation and erosion.

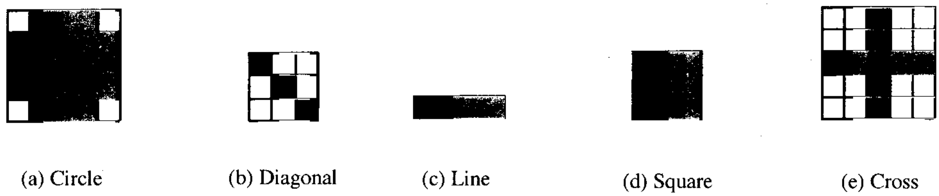


Figure 3. Some possible structuring elements.

3.2. *The hit_miss and other operators*

Based on these two basic operators, i.e., dilation and erosion, a number of new operators have also been developed, such as closing, opening, hit_miss, conditional dilation, conditional erosion, sequential dilation, conditional sequential dilation, and so on. The *opening* ($\circ$) and *closing* ($\bullet$) operators are very suitable for the manipulation of area features. These two operators are defined as follows:

$$\text{Open:} \quad A \circ B = (A \ominus B) \oplus B \tag{2a}$$

$$\text{Close:} \quad A \bullet B = (A \oplus B) \ominus B. \tag{2b}$$

The hit_miss operator is the basic tool for shape detection and construction in morphological algorithms. It will also be used in the models described in this paper. It is defined as follows:

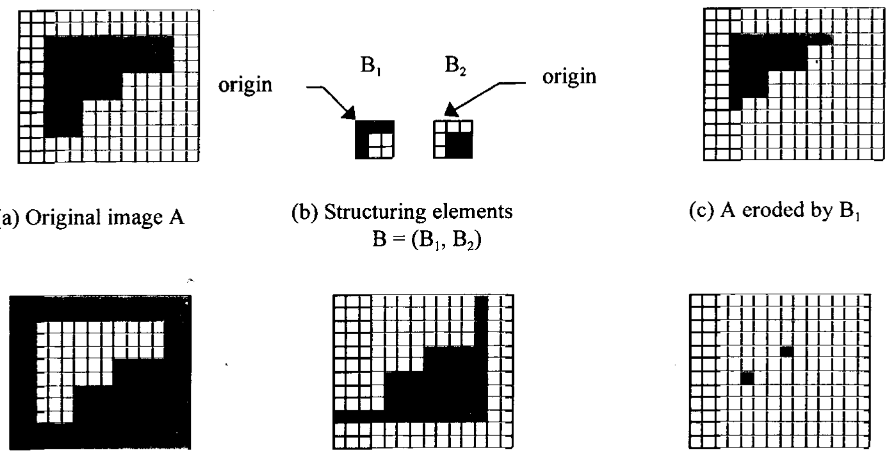
$$A \otimes B = (A \ominus B_1) \cap (A^c \ominus B_2) \tag{3}$$

where A is an original feature, A^c is a complement (or background) of A, B is a structuring element pair, i.e., $B = (B_1, B_2)$, one to probe the inside and one to probe the outside of the feature. A point is obtained in the hit_miss output if and only if B_1 translated to the point fits inside A and B_2 translated to the point fits outside A. It is assumed B_1 and B_2 are disjoint, i.e., $B_1 \cap B_2 = \varnothing$. The process of the hit_miss operator is illustrated in figure 4.

In figure 4, the hit_miss operator is used to detect and extract a feature with a "Γ" shape. Figure 4(a) shows the original feature and figure 4(f) shows the result. The hit_miss operator is so important that it is involved in most morphological algorithms.

4. Modified skeletonization algorithm for area-to-line collapse

The development of skeletonization algorithms has been one of the main topics in the field of mathematical morphology. Several of them have been published [19], [20], [24], [25]. They are based on three different approaches, i.e., homotopic thinning and homotopic sequential thinning [19], crest-line of distance functions [15] and contours [1]. The first approach is the most fundamental one and is employed in this study.



(a) Original image A
(b) Structuring elements $B = (B_1, B_2)$
(c) A eroded by B_1
(d) A^c: Complement of A
(e) A^c eroded by B_2
(f) Final result: (c) $\cap$ (e)

Figure 4. Process of hit_miss operator.

4.1. Thinning-based algorithm for deriving skeleton

The thinning-based skeletonization algorithms consist of two steps: thinning and pruning. The first is for deriving a general skeleton of an area feature, the second is for removing parasitic branches of the skeleton derived by the thinning algorithm.

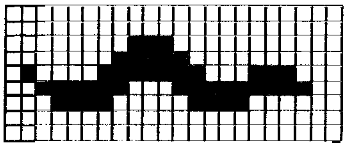
The thinning of feature A by a structuring element B can be described by Equation (4).

$$A \bigcirc B = A - (A \otimes B) = A \cap (A \otimes B)^c. \tag{4}$$

Where, A is an original image and B is a structuring element. The symbols "$\bigcirc$", " $-$ ", "$\otimes$", "$\cap$" respectively stand for thinning, set difference, hit-miss and set intersection. A^c stands for complement of set A.

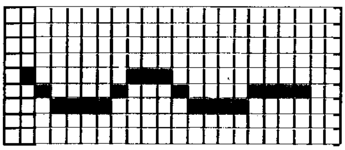
The thinning process is illustrated in figure 5. It is clear that, as the name implies, the thinning process makes the area thinner, i.e., the feature shown in figure 5(d) is thinner than the original feature shown in figure 5(a). The feature shown in figure 5(d) is not yet the skeleton of the feature shown in figure 5(a). To obtain a skeleton, the thinning process needs to be repeated so that the final result is a line feature with a width of only one pixel. In practice, in this repeated process, not a single structuring element is used at all times but a set of structuring elements is used sequentially, instead. In other words, the more useful expression for symmetrically (more than once) thinning a feature, A, is based on a sequence of structuring elements which are described by Equation (5) and illustrated in figure 6.

$$\{B_i\} = \{B_1, B_2, B_3, \ldots B_{n-1}, B_n\}. \tag{5}$$

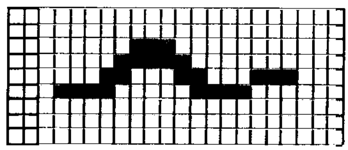


(a) Original feature A

(b) Structuring element B



(c) A ⊗ B



(d) A - (A ⊗ B)

Figure 5. Process of thinning operation.

Figure 6. Sequence of structuring elements for systematic thinning (''x'' means ''don't care'').

Using this sequence of structuring elements, the process of a systematic thinning can be described by Equation (6) [19]:

$$A \bigcirc \{B_i\} = ((\ldots((A \bigcirc B_1) \bigcirc B_2)\ldots) \bigcirc B_n). \tag{6}$$

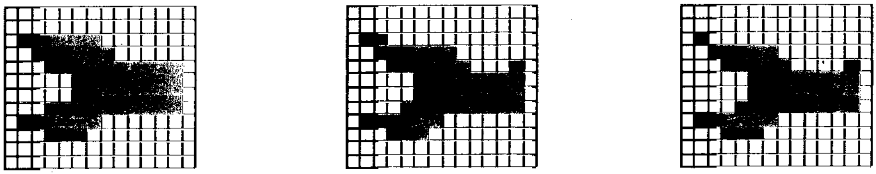
The sequence of structuring elements shown in figure 6 is called homotopic structuring elements [5]. Accordingly, the algorithm described by Equation (6) is named ''homotopic sequential thinning algorithm''. [Homotopic sequential thinning is a type of homotopic transformation which has the characteristics of the retention of the original topology.]

The homotopic sequential thinning process described in Equation (6) is to thin A by one pass with B_1, then the result with one pass of B_2, and so on, until A is finally thinned with one pass of $B_2 \to B_n$. For skeletonization, the entire process is repeated again and again until no changes occur, i.e., when lines with only one pixel width are obtained. This process is illustrated in figure 7.

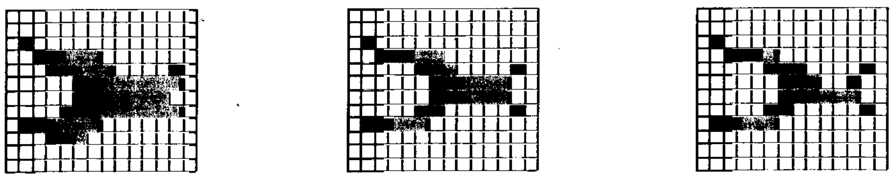
Using $S_k(A)$ to denote the skeleton of feature A, the skeletonization using this homotopic sequential thinning algorithm can be written as follows:

$$S_k(A) = A \bigcirc \{B_i\}. \tag{7}$$

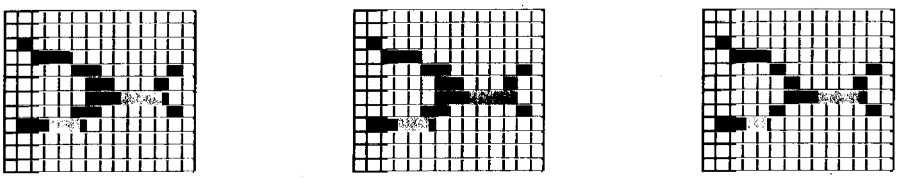
As shown in figure 7(i), some parasitic branches still exist in the result obtained from this thinning-based skeletonization. In the case of figure 7(i), the parasitic branches are on the



(a) Original feature A   (b) $T_1 = A \bigcirc B_1$ (c) $T_2 = T_1 \bigcirc B_2$



(d) $T_3 = T_2 \bigcirc B_3$ (e) $T_4 = T_3 \bigcirc B_4, B_5, B_6, B_7, B_8$ (f) $T_5 = T_4 \bigcirc B_1$



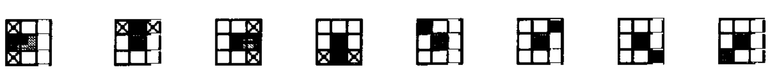
(g) $T_6 = T_5 \bigcirc B_2$ (h) $T_7 = T_6 \bigcirc B_3, B_4, B_5, B_6, B_7, B_8$ (i) $T_8 = T_7 \bigcirc B_1$

Figure 7. Homotopic sequential thinning algorithm for skeletonization.

rightmost and leftmost part of the skeleton. They need to be removed. This removal process is called "pruning", which can be described by Equation (8).

$$P(X) = X \bigcirc \{P_i\}. \tag{8}$$

The form of this equation is essentially identical to the homotopic sequential thinning algorithm defined by Equation (6). The only difference is that a set of specially designed structuring elements as shown in figure 8, $\{P_i\}$, is used in this process, so that this approach is capable of pruning parasitic branches in the eight basic directions, i.e., right, left, up, down, upper-right, upper-left, lower-right and lower-left. The function of each structuring element is listed as table 1.



(a) (b) (c) (d) (e) (f) (g) (h)

Figure 8. Sequential structuring elements for pruning parasitic branches ("x" means "don't care").

Table 1. Functions of structuring elements shown in figure 8.

Structuring Element	*(a)*	*(b)*	*(c)*	*(d)*	*(e)*	*(f)*	*(g)*	*(h)*
Direction of effect	Right	Lower	Left	Upper	Lower-right	Lower-left	Upper-left	Upper-right

The effect of pruning is achieved by applying Equation (8) repeatedly N times with this set of structuring elements. For example, the parasitic branches shown in figure 9(a) are removed after applying this algorithm twice and the result is shown in figure 9(b). Now the determination of the value for ''N'' becomes the critical problem.

4.2. Modification of skeletonization algorithm for area-to-line collapse

As discussed in the previous sub-section, by applying the two morphological algorithms in a sequence, i.e., homotopic sequential thinning and pruning, the skeleton of areal features can be derived. However, there is no solution for automated control over loop number N in the pruning process expressed by Equation (8).

Through a number of experiments, it has been found that it is possible to overcome this defect by using a subset of $\{P_i\}$ shown in Equation (8) and by integrating the thinning and pruning processes. More exactly, in this modification, only the last four structuring elements in $\{P_i\}$ are selected and the complete skeletonization algorithm for the area-to-line collapse is in the form of Equation (9).

$$S_k(A) = A \bigcirc \{B_i\} \bigcirc \{E_i\} \tag{9}$$

where $\{B_i\}$ is the same as Equation (5) and shown in figure 6, and $\{E_i\}$ includes the last four structuring elements from $\{P_i\}$, i.e., the fifth to eighth ones shown in figure 8. In this way, the entire skeletonization process can be controlled automatically, i.e., it stops when no further changes occur if the process is repeated.

To illustrate the effectiveness of this modified algorithm, two simple examples as shown in figure 10 are given here. Figure 10(a) presents two original features, figure 10(b) shows

(a) Original feature

(b) Result

Figure 9. Effect of pruning process.

the results after applying the skeletonization algorithm without the pruning process and figure 10(c) illustrates the results after applying the pruning process.

5. Morphological models for partial collapse operation

In the previous section, a modification is made of a skeletonization algorithm for area-to-line collapse. This algorithm will also form a basis for the partial collapse operation, which is discussed in this section.

5.1. Approach followed

As discussed in Section 2, partial collapse occurs when part or parts of an area feature are long but thin. The main tasks of this operation are (a) to make these long but thin features collapse completely, i.e., from area to line, and (b) to simplify the remaining part of the area feature. Therefore, for the partial collapse operation, the critical issue is how to integrate area-to-line collapse and area simplification operations in a logical manner.

The line of thought followed in this study is (a) those areas which are thinner than a certain value (or an objective criterion) will be eroded using an erosion operation, (b) those thin parts which are eroded will then be replaced by the skeletons of themselves, and (c) the main part of the area which remains after erosion will be simplified, i.e., its boundary is smoothed. This basic concept can be illustrated in figure 11.

5.2. Morphological procedure for partial collapse

In order to make the outline described above work, several other processes also need to be involved. Using morphological operators, this process can be implemented by a four step procedure as follows:

(a) Derive the skeleton of an area feature A using Equation (10).

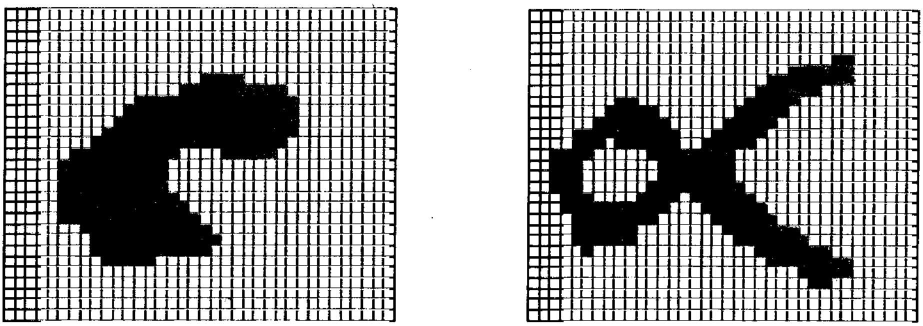
$$S_k(A) = A \bigcirc \{B_i\} \bigcirc \{E_i\}. \tag{10}$$

(b) Separate the areal part from the whole feature A using an opening operator with a circular structuring element S_1, where the size of S_1 is the width of the linear part in terms of pixels plus 1.

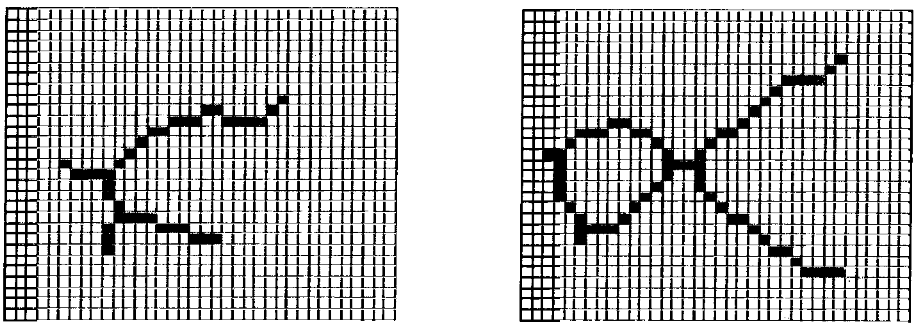
$$C = A \circ S_1. \tag{11}$$

(c) Simplify the areal feature C using closing and opening operators, where S_2 is a critical value which determines which parts are to be collapsed.

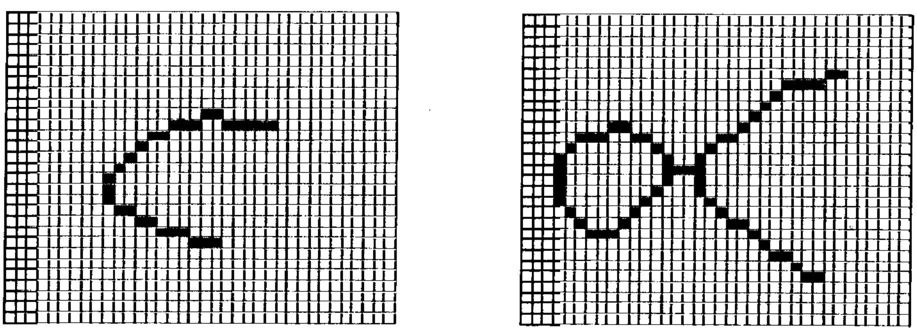
$$Z = (C \bullet S_2) \circ S_2. \tag{12}$$



(a) Original features



(b) Result without pruning process



(c) Result with pruning process

Figure 10. Morphological skeletonization process for irregular features.

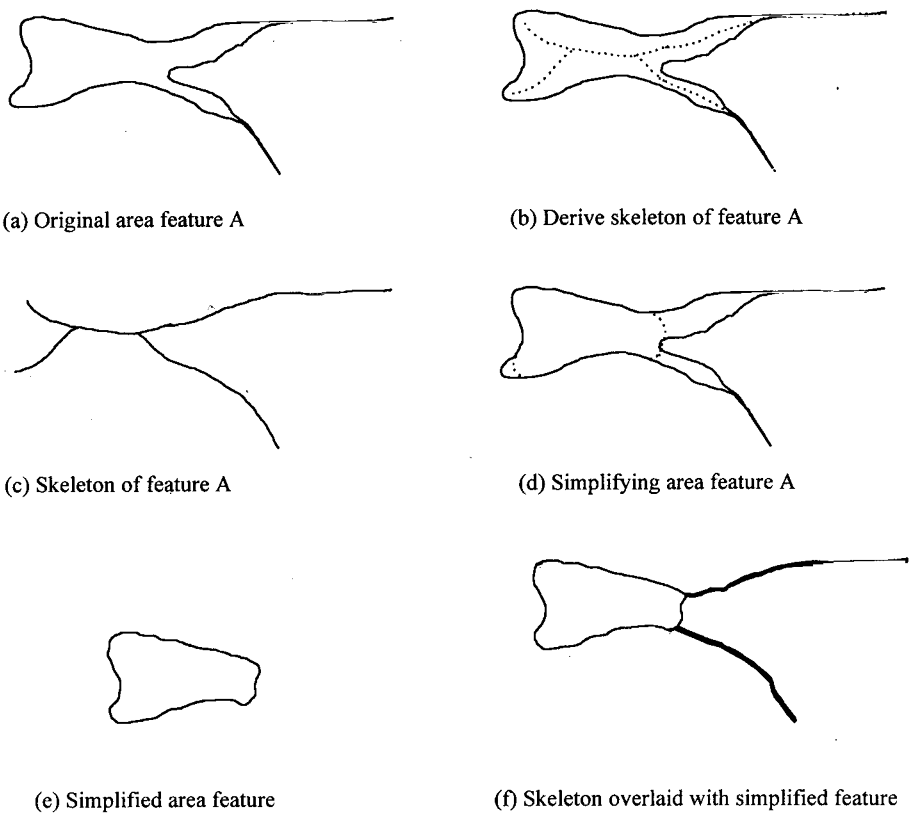


Figure 11. Process of partial collapse operation.

A discussion of S_2 will be conducted at a later stage.

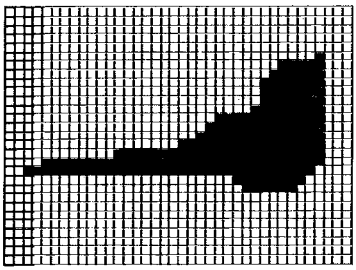
(d) Obtain final result by overlaying $S_k(A)$ and Z. In this sub-process, the width of the skeleton is one pixel, therefore a dilation operator could be required to thicken the skeleton so that it can be represented at smaller scale.

Figure 12 illustrates this partial collapse operation for various scale reduction factors. (In this illustration, pixel size is assumed to 1.5 mm and D_c (described below) is assumed to be 1 pixel size.)

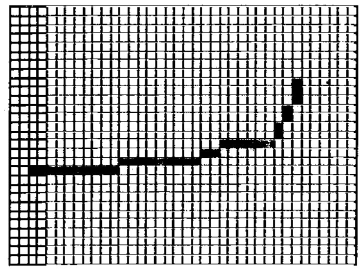
The value of S_2 which is a critical value for partial collapse can be calculated by Equation (13)

$$S_{2\_size} = \frac{S_{target}}{S_{source}} \times D_c \tag{13}$$

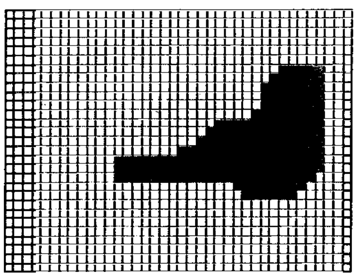
where $1{:}S_{source}$ and $1{:}S_{target}$ are the scales of the source and target data respectively. D_c is



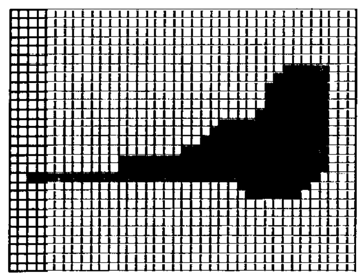
(a) Original Feature A



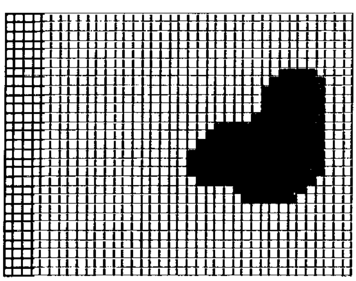
(b) Skeleton of A



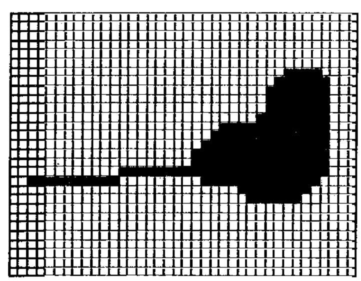
(c) A closed and opened by S (3 × 3) suited for representation at 2× reduction



(d) Overlap (b) and (c) suited for representation at 2× reduction



(e) A closed and opened by S (5 × 5) suited for representation at 4× reduction



(f) Overlaid (b) and (e) suited for representation at 4× reduction

Figure 12. Morphological transformation for partial collapse operation.

the threshold of the width of an area feature in terms of the number of pixels, below which that part of the feature on the target map should be collapsed. This threshold is called the *threshold of collapse* in this study and it should more or less equal the minimum visible size—about 0.7 mm at target map scale [17].

If a symmetric structuring element with the origin at the centre is to be used, then the dimension of the structuring element should be an odd number. In this case, Equation (13) can be written as follows, where INT means the integer part of the value:

$$S_{2\_size} = INT\left(\frac{INT\left(\frac{S_{target}}{S_{source}} \times D_c + 0.5\right)}{2}\right) \times 2 + 1. \quad (14)$$

6. Experimental testing

In the previous two sections, morphological models for both area-to-line collapse and partial collapse operations were illustrated. These models are tested in this section to see whether they work well with real map data sets. Since the skeletonization algorithm can be used either as a complete model for area-to-line collapse purposes or as a component in the model for partial collapse, no separate test on the area-to-line operation will be conducted.

6.1. Test results

A reservoir with a river network, as shown in figure 13(a), is selected for this test. It is at 1:20,000 map scale, from an Australian Map Atlas, and converted to raster format with a resolution of 150 dpi. Results of partial collapse for four smaller scales, i.e., 1:40,000, 1:100,000, 1:160,000 and 1:200,000 are produced. With the given S_{target}/S_{source} ratios of 2, 5, 8 and 10, the size of the structuring elements used in a closing and an opening operator, according to Equation (14), are 9, 21, 33 and 41 pixels respectively. Circular structuring elements are considered in this test.

Figures 13 to 17 show both the intermediate and final results. Figure 13(b) is the skeleton of the reservoir with line width of one pixel and obtained by step (a). Figure 13(c) shows the areal part which is separated from figure 13(a) at step (b). Figures 14 to 17 show the process and results for partial collapse of this reservoir for representation at 1:40,000, 1:100,000, 1:160,000 and 1:200,000, respectively. In these figures, (a) and (b) represent the application of Equation (12) to simplify the areal part of the feature; (c) is the representation of overlaying the areal part with the skeleton; and (d) is the result at the smaller scale. For figures 14 to 17, the skeleton of the whole feature (see figure 13(b)) has been respectively dilated by $3 \times 3, 5 \times 5, 9 \times 9$ and 11×11 structuring elements in order to provide a suitable skeleton at different target scales.

It should also be noted that this example mainly concerns the partial collapse of area features, but not the generalization of a river network as such. Therefore, some shorter

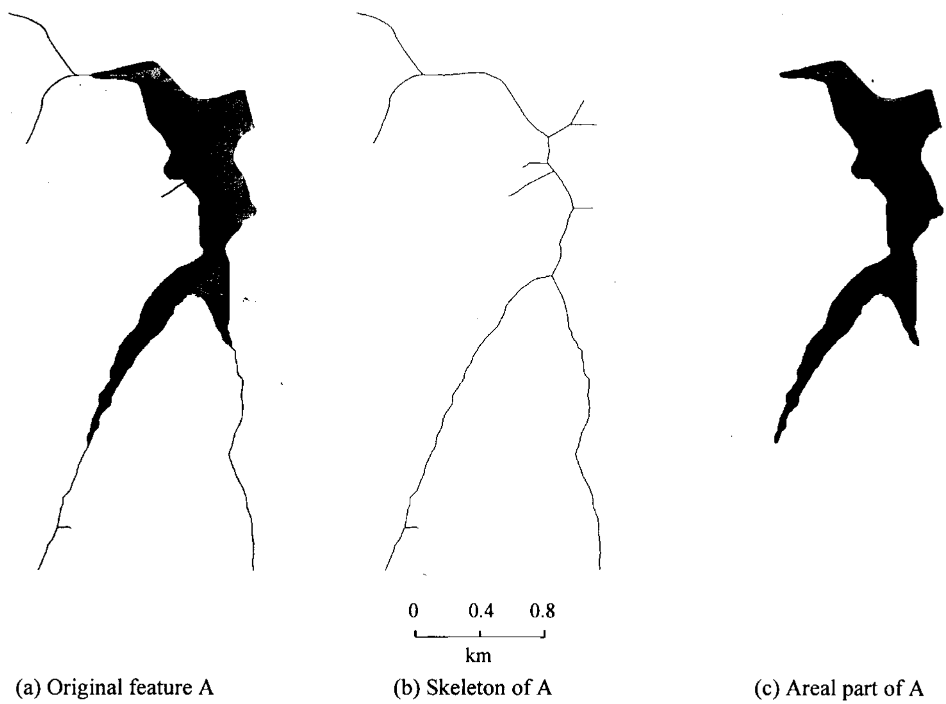


Figure 13. Victoria Reservoir. (Original map scale is 1:20,000, graphics display reduced by 50%.)

branches which should be omitted are not eliminated, e.g., L_1 and L_2 in both figures 16 and 17.

6.2. *Discussion*

In the model for area-to-line collapse using the modified skeletonization algorithm, the twelve structuring elements illustrated in figure 6 and figure 8(e), (f), (g) and (h) play an important role in the morphological skeletonization. The test results show that this model is good at deriving the skeleton of individual area features. However, some undesirable results can occur at junctions when this algorithm is applied to combined area features. A simple example is shown in figure 15. It shows that this algorithm may produce unacceptable results if it is to be applied to derive the skeleton of a road network. This is not the fault of the skeletonization algorithm itself. Instead, it is due to the definition of the skeleton. In this case, some constraints may need to be enforced [20].

In the morphological model for the partial collapse operation, two major transformations are involved: skeletonization and area simplification. The latter is realized through a close and an opening operator. In some particular cases, after applying an opening operator, an area feature may be separated into two or more features if there are

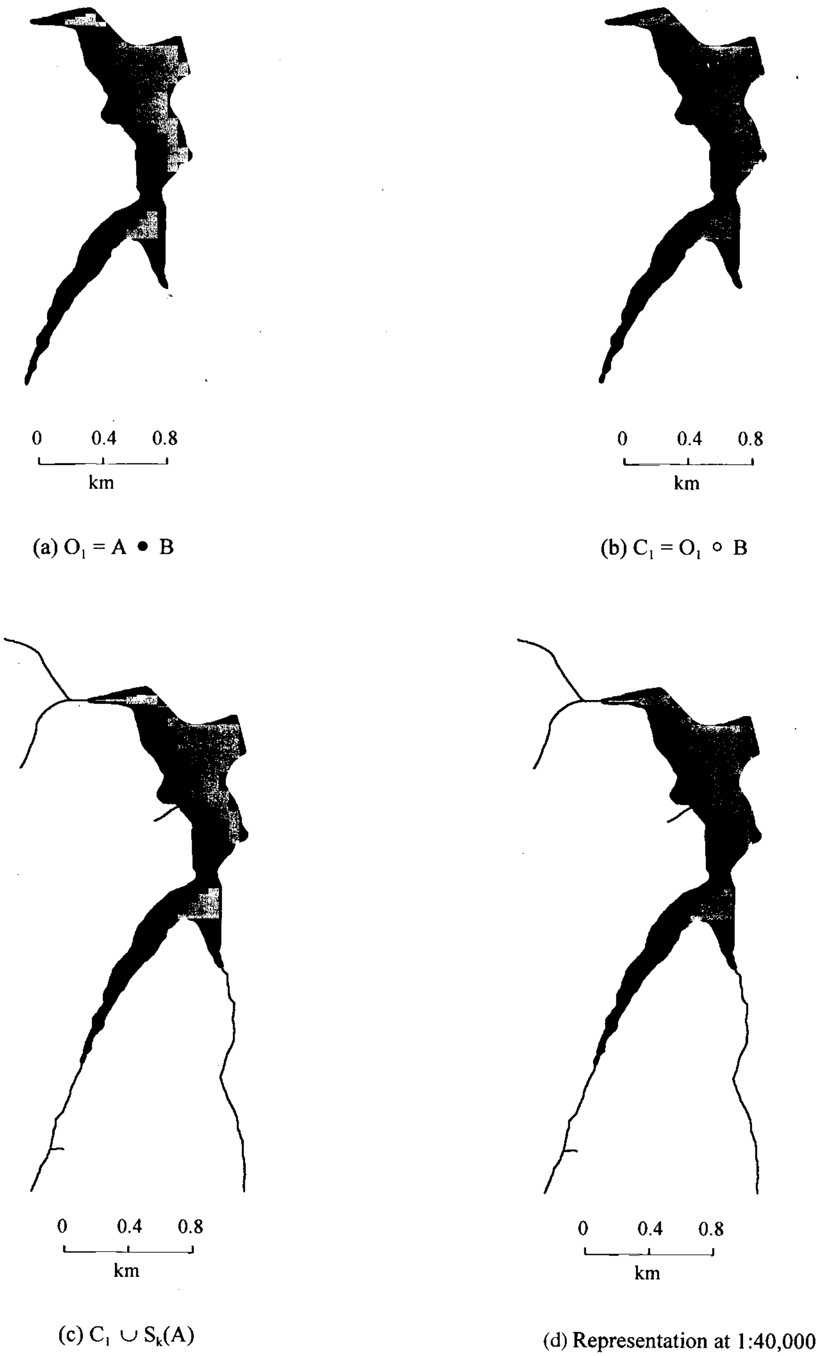


Figure 14. Victoria Reservoir suited for representation at 1:40,000. (*B* is 9×9 circular structuring element.)

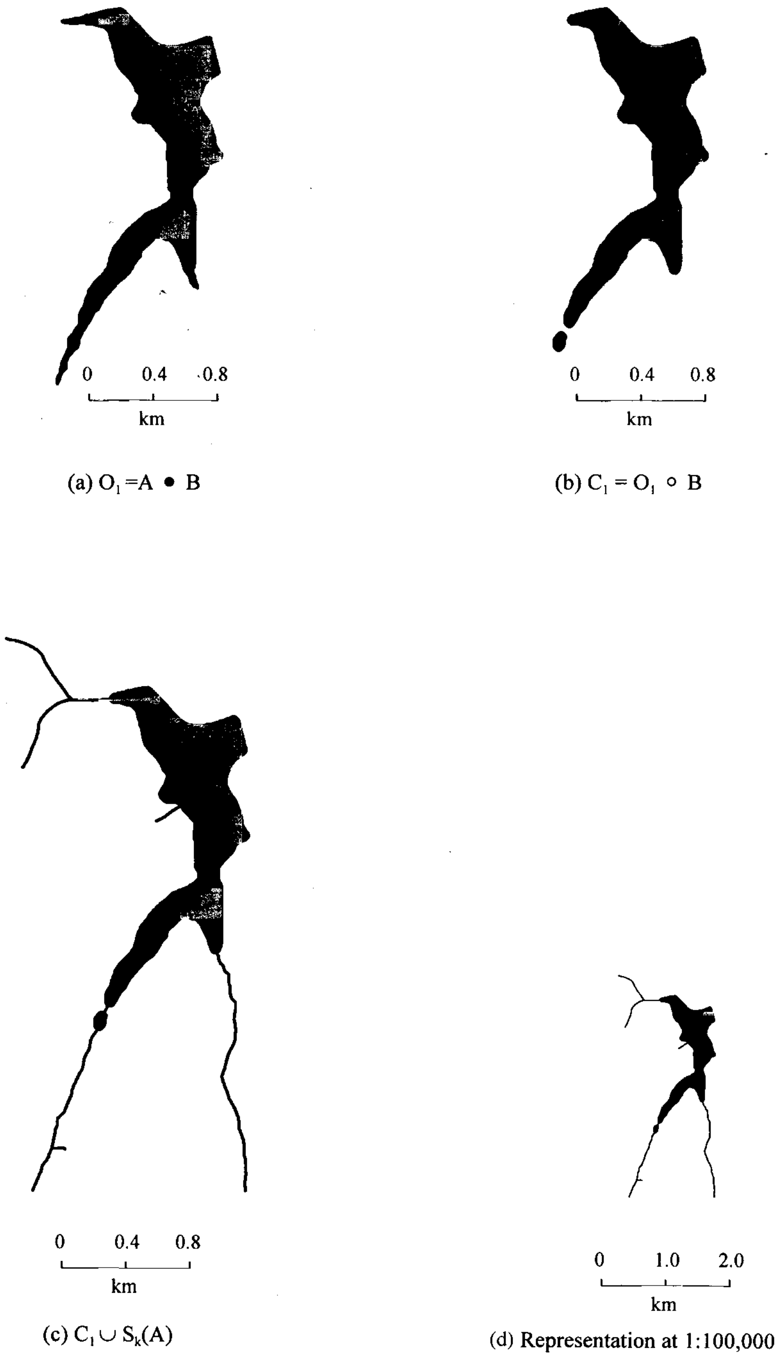


Figure 15. Victoria Reservoir suited for representation at 1:100,000. (*B* is 21 × 21 circular structuring element.)

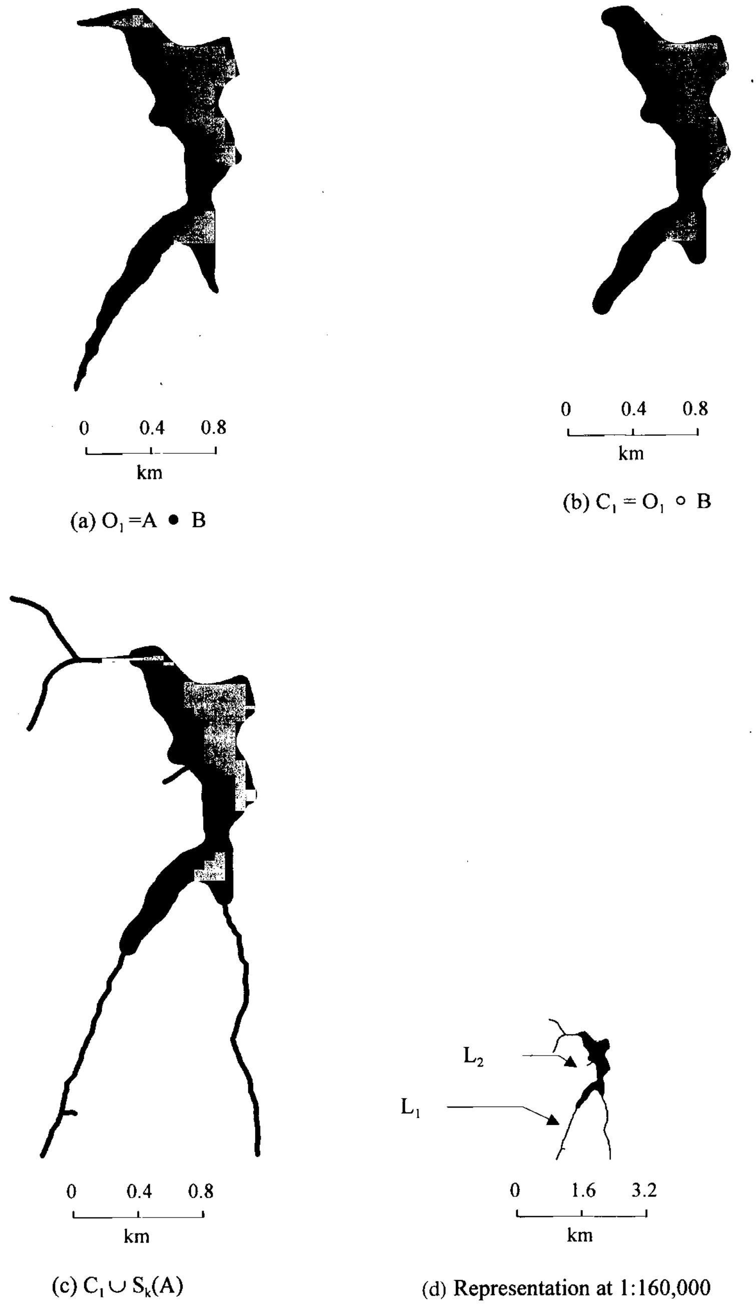


Figure 16. Victoria Reservoir suited for representation at 1:160,000. (B is 33×33 circular structuring element.)

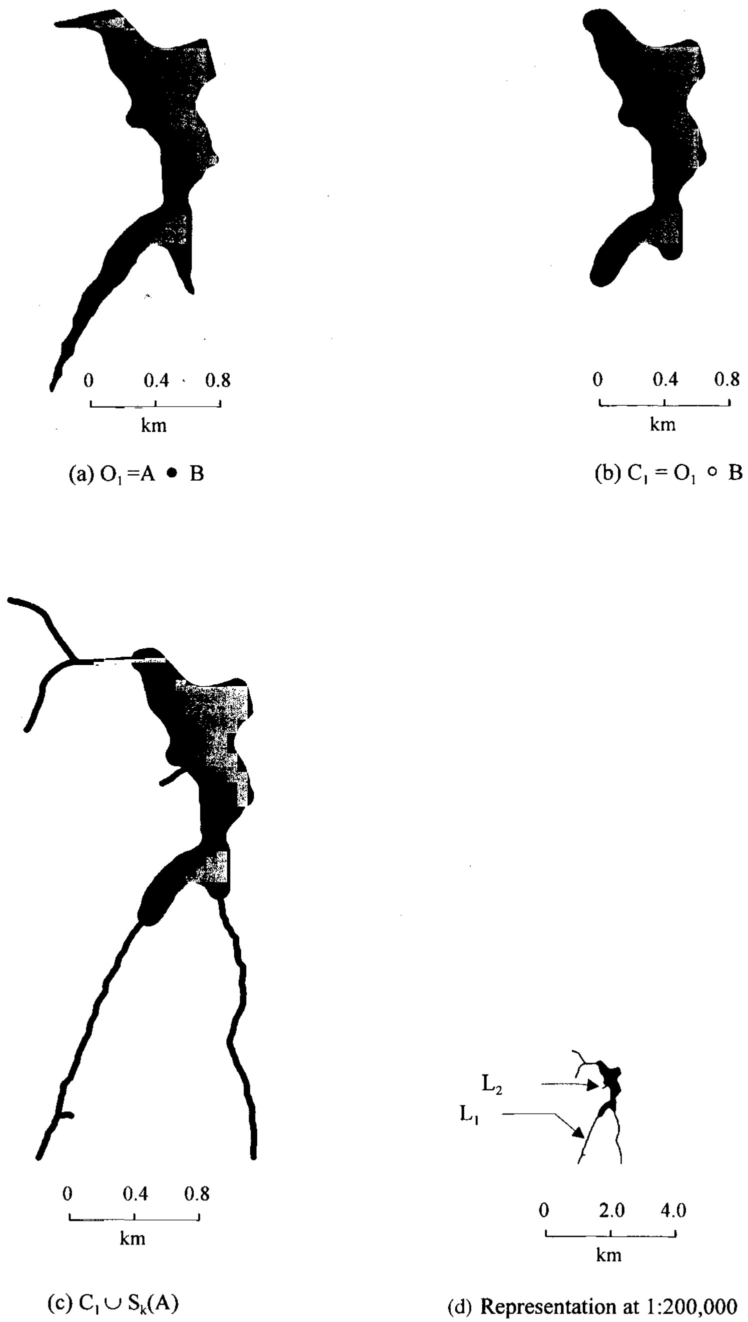
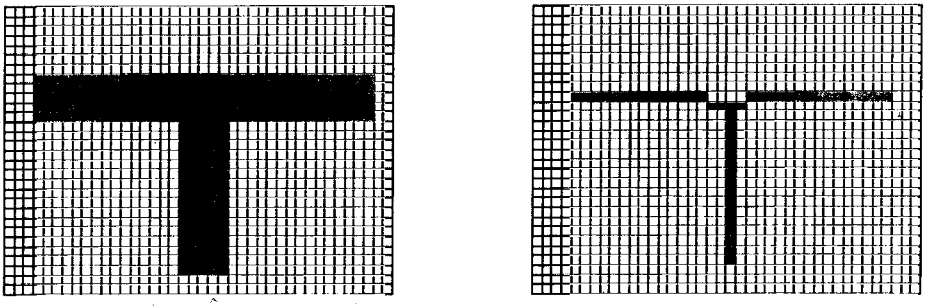


Figure 17. Victoria Reservoir suited for representation at 1:200,000. (B is 41 × 41 circular structuring element.)



(a) Original features (b) Result of pruning process

Figure 18. Morphological skeletonization process for regular features.

some narrow necks. This can be seen in the transformation process from 1:20,000 to 1:100,000 shown in figure 15(c) which is identical to figure 19(a), where the reservoir is separated into two parts which are connected by a single line feature after a scale reduction. However, if one considers that the smaller areas should not be represented at that scale map, an optional post-processing could be applied to eliminate them, as demonstrated in figure 19, such as a morphological elimination process [23].

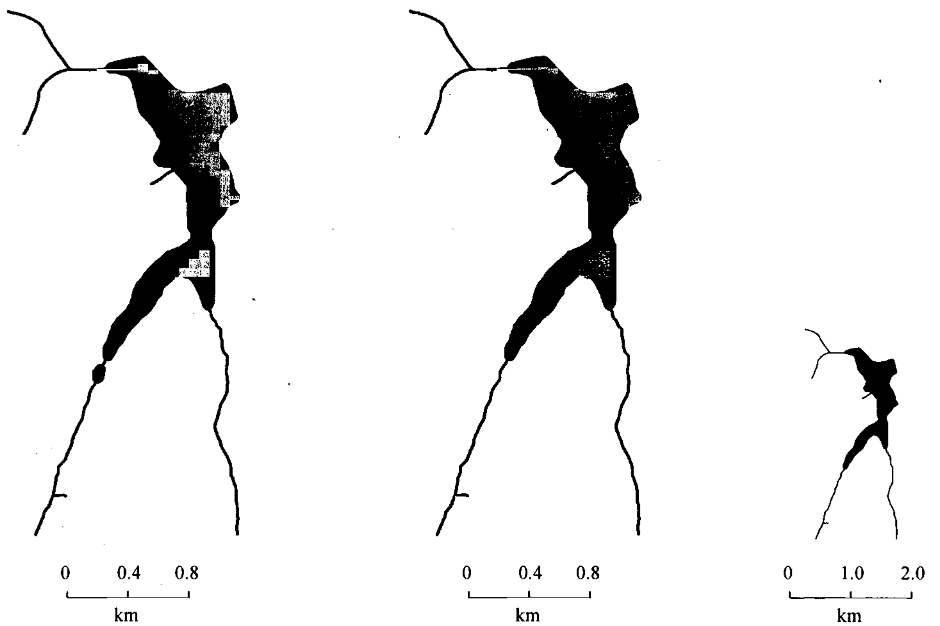
7. Conclusions

In this paper, morphological models for area-to-line collapse and partial collapse operations have been developed. The area-to-line collapse operation is realized through the morphological skeletonization algorithm, which is modified from the homotopic sequential thinning algorithm. The modification was made in order to have automated control over the pruning process. Thus, the entire skeletonization process stops when no further changes occur if the process is repeated.

The partial collapse operation mainly involves two transformations: area-to-line collapse and area simplification. In other words, models for the partial collapse are a result of integration of these two transformation processes. The procedure for such integration is a contribution by the authors although the basic morphological operators are well established. The area simplification is a scale-driven process and is realized by a close and an opening operator with the same structuring element. Thus, this is a quite objective process.

These models have been tested using map data sets. In the evaluation of experimental results, two problems were faced by the authors which were: (a) there is no reported previous study of partial collapse in a digital environment, and (b) the results by human operators are very subjective for this particular operation. Nevertheless, the authors consider that the results are quite reasonable and acceptable.

The techniques used in this paper are in raster mode. If the original data is in vector format, then one needs to (a) rasterize the vector data, (b) perform collapse operation; and



(a) Result generated for representation at 1:100,000

(b) Smaller part is eliminated

(c) Representation at 1:100,000

Figure 19. Optional post-processing for partial collapse operation.

(c) vectorize the resulting features. The accuracy loss due to vector/raster/vector conversion can be controlled easily. The speed of such processes in raster mode is largely dependent on raster size and the magnitude of scale change that determines the sizes of structuring elements to be used. Research into the methodology (e.g., decomposition of structuring elements) for speeding up such operations is also being conducted.

It must finally be pointed out that this paper does not aim to describe a system for the collapse operation but rather a mathematically elegant solution, models and algorithms for this operation. It is hoped that the models presented in this paper will form a mathematical basis for an objective collapse operation, especially the partial collapse. Such models might be compared to other transformation models, such as affine and conformal models and form the foundation of generalization. Before such algorithms (or models) have been developed for the full range of generalization operations, as one can imagine, it is difficult, if not impossible, to implement a fully automated system. At this stage, an interactive system using amplified intelligence—an idea proposed by Weibel [27]—is more realistic. As the nature of this paper is about algorithm development but not system development, therefore, detailed discussions about how to develop such systems lie outside the scope of this paper.

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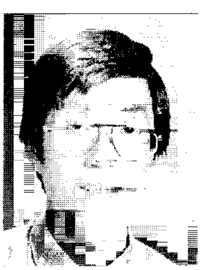

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