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Basic Topological Models for Spatial Entities in 3-Dimensional Space

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Abstract

In recent years, models of spatial relations, especially topological relations, have attracted much attention from the GIS community. In this paper, some basic topologic models for spatial entities in both vector and raster spaces are discussed.

It has been suggested that, in vector space, an open set in 1-D space may not be an open set any more in 2-D and 3-D spaces. Similarly, an open set in 2-D vector space may also not be an open set any more in 3-D vector spaces. As a result, fundamental topological concepts such as boundary and interior are not valid any more when a lower dimensional spatial entity is embedded in higher dimensional space. For example, in 2-D, a line has no interior and the line itself (not its two end-points) forms a boundary. Failure to recognize this fundamental topological property will lead to topological paradox. It has also been stated that the topological models for raster entities are different in Z^2 and R^2. There are different types of possible boundaries depending on the definition of adjacency or connectedness. If connectedness is not carefully defined, topological paradox may also occur. In raster space, the basic topological concept in vector space—connectedness—is implicitly inherited. This is why the topological properties of spatial entities can also be studied in raster space. Study of entities in raster (discrete) space could be a more efficient method than in vector space, as the expression of spatial entities in discrete space is more explicit than that in connected space.

Keywords: topological models, topological properties, vector space, raster space, spatial entities

1. Introduction

In daily life, people are concerned with information about spatial entities (or objects), such as their positions and attributes, and/or the spatial relations between them, such as direction, distance, adjacency, topology and so on. Spatial entities, after being abstracted into mathematical entities, can then be studied in terms of algebra, order and topology. Algebraic properties are related to the concepts of direction and distance. Topological properties are related to distance and topological relations. Order properties are related to comparison between any two entities.

Topological information is related to the neighborhood of spatial entities and is of particular interest to the GIS community, as the topological properties of spatial entities are the most fundamental, compared with others. In GIS literature, a body of literature on topological relations can be found. The most widely used model for the description of topological relations is the 4-intersection model developed by Egenhofer and Franzosa in 1991 [1]. In this model, the topological relations between two entities A and B are defined in terms of the intersections of A's interior and boundary with B's interior and boundary. Egenhofer and his collaborators have since extended this model into a 9-intersection model by introducing another element of a spatial entity, i.e., the exterior, which is then represented by its complement. However, it has been realized that there is a topological paradox introduced by the definitions of these topological concepts used in the 9-intersection model [3]. It is then of interest to undertake a close examination of this paradox.

Spatial entities are represented either in vector format or in raster format, and there has been an almost constant debate about their natures and their importance in the future, throughout the history of GIS [8], [9], [10]. To discuss the topology of spatial entities, some fundamental issues on topology both in vector space and in raster space need to be discussed. Indeed, this is the topic of this paper.

This introduction is followed by a discussion of some basic concepts in topology (Section 2), then some topological issues in vector space (Section 3) and raster space (Section 4) are examined. After that, the topological relationships between vector space and raster space are discussed.

2. Topological foundation in vector space

2.1. The foundation of vector space

To describe mathematical entities, which are abstracted from spatial entities, one has to employ a referencing system. The Cartesian coordinate system is the most widely used one. As it is a system, there must be some hardware and software. The hardware of the system is a set (or universe U) and the software is comprised of some unary, binary, 3-ary (or ternary) ... and N-ary relations ... on this universe. For example, in a two-dimensional plane coordinate system, the R^2 (R^2 is the Cartesian product of R and R, where R is a real set), is the hardware. The " $+$ ", " $-$ ", " $\times$ ", " $-$ " between two elements and scalar multiplication, which can be viewed as a 3-ary relation on R^2, are the software in R^2. Thus, the R^2 with such an algebraic structure is called vector space or linear space and is denoted as IR^2.

In IR^2, one can then discuss the direction of a straight line, the angle between two straight lines, linear transformations and so on. However, one still cannot discuss topological properties and topological relations.

2.2. The topological structure of vector space

In order to make it possible to discuss the topological properties and topological relations of spatial entities, one has to define the so-called topological structure. This is established with the following concepts as its foundation: distance, circle, inner points and open set.

A topological structure simply consists of a set (U) and a specified family of subsets ($\hbar$) called open sets, which satisfy some conditions. This structure is then denoted as $\langle U, \hbar \rangle$ and satisfies the following three conditions [5]:

Axiom 1: Any union of open sets is open;

Axiom 2: The intersection of any two open sets is open; and

Axiom 3: $\varnothing$ and U are open.

Particularly, $\langle R, \Re \rangle$, $\langle R^2, \Im \rangle$ and $\langle R^3, \aleph \rangle$ are the usual topological structures of vector space, which are induced by Euclidean distance. For simplicity, late on in this paper, the term "vector space" simply refers to the vector space with topological structure. 1-D, 2-D and 3-D vector spaces with such topological structure are still denoted as IR, IR^2 and IR^3 respectively. Here R, R^2 and R^3 can be viewed as hardware. $\Re$ is a family of subsets of R, each element of which meets some specified the three axioms above and can be viewed as a unary relation on R. Similarly, $\Im$ is a family of subsets of R^2, and $\aleph$ is a family of subsets of R^3. From the system point of view, $\Re$, $\Im$ and $\aleph$ can be viewed as software on these hardware platforms, i.e., R, R^2 and R^3. Both software and hardware together form a system. After its formation, the system will then have functions. In other words, after the definition of the topological structure for IR, IR^2 and IR^3, the topological properties and relations of spatial entities in IR, IR^2 and IR^3 can then be discussed.

3. Topological properties of spatial entities in vector space

After the definition of the topological structure, the topological properties and relations of spatial entities can then be discussed. In geo-information science, most discussions are still on 1-dimensional and 2-dimensional representations of spatial entities in vector space. Therefore, in this section, 3-D spatial entities will not be discussed. In this discussion, some topological terms are adopted from the textbook by Henle [4].

3.1. The basic topological components of spatial entities in vector space

The basic topological components of a spatial entity are interior, boundary and exterior. In one-dimensional vector space (IR), the basic geometric characteristic of all elements of $\Re$ is that they are straight lines. The basic topological components of straight lines are illustrated in figure 1. However, in 2-dimensional space IR^2, the basic geometric characteristic of the elements of $\Im$ is that they are areal disks. The basic topological concepts of a spatial entity in IR^2 are illustrated in figure 2.

It is clear that in both 1-D and 2-D space, the fundamental topological property of spatial entities is that the boundary of a spatial entity separates its interior from its exterior. It is the same for 3-D space. Indeed, this is a topological convention.

In 2-D, this topological property is described by the so-called "Jordan Curve" which is illustrated in figure 3. In this figure, L represents the boundary of the area, I and E denote its interior and exterior, x denotes a point in its interior, and y denotes a point in its exterior.

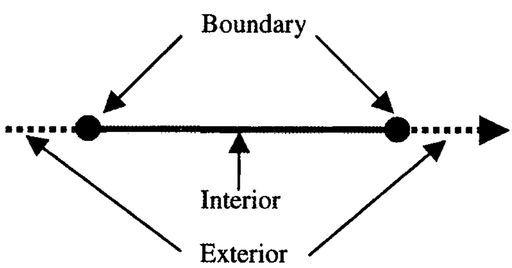


Figure 1. Topological concepts of spatial entities in 1-D (IR) vector space.

If one wants to travel from x to y, one must cross the boundary at least once. In other words, ''A closed curve separates its interior from its exterior''. This curve is called the Jordan Curve.

3.2. Topological paradox associated with dimensionality in vector space

As discussed in the previous sub-section, there are 1-D and 2-D spatial entities in vector space and each entity has an interior, a boundary and an exterior. The basic topological convention is that the boundary of a spatial entity separates its interior from its exterior. However, these definitions of topological concepts are associated with a dimension. This topological convention holds to be true only in a space with a particular dimension. Otherwise, topological paradoxes will appear. For example, if a 1-D entity is embedded in 2-D (a higher dimension) space while the same definitions in 1-D are simply adopted, then a paradox appears. This is illustrated in figure 4, where the interior of an entity meets its exterior. This is also the case if a 2-D entity is embedded in 3-D space while the same definitions in 2-D are simply adopted.

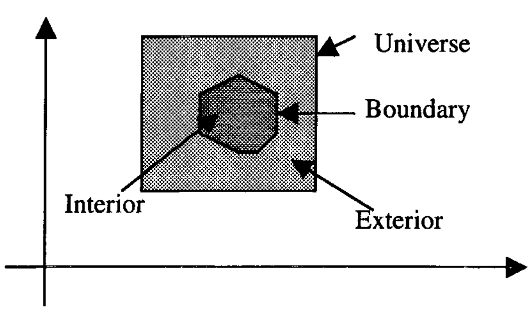


Figure 2. Topological concepts of spatial entities in 2-D (IR^2) vector space.

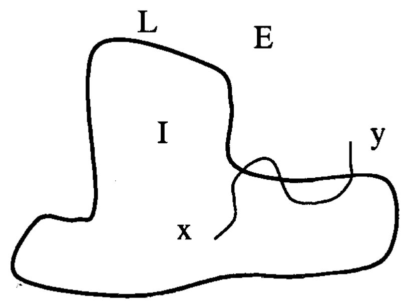


Figure 3. Jordan's curve in vector space: A closed curve separates its interior from its exterior.

3.3. The concept of "open set" in multi-dimensional vector space

This topological paradox has also been realized by other researchers [2]. The questions arising are "Why and how this happens?" and "How to solve this problem?" Indeed, these two questions are inter-related. If the first question can be answered, then the second question will be easy to answer. To answer these questions, another fundamental concept of topology—the open set—needs to be introduced and discussed.

From the discussions in the previous section, it is clear that the union of its interior and boundary form a spatial entity itself. This union is called a "closure" in topological terms. Opposite to closure is the concept of "open".

A set is considered as an "open set" if, for every point in this set, there is always an open neighborhood contained by the set. In *IR*, an open neighborhood is a line segment. Given a line segment, say *AB*, for every point of *AB* there is always an open neighborhood contained by *AB* (figure 5a). In IR^2, an open neighborhood is an areal disk. Given an areal disk, say *D*, for all points of *D* there is always an open neighborhood contained by *D* (figure 5b). In these cases, *AB* is an open set in 1-D space and *D* is an open set in 2-D space.

However, if a line (a 1-D spatial entity) is embedded into a 2-D space, then a problem arises. In IR^2, given a line segment, say *AB*, it is impossible to find an areal disk (i.e., an

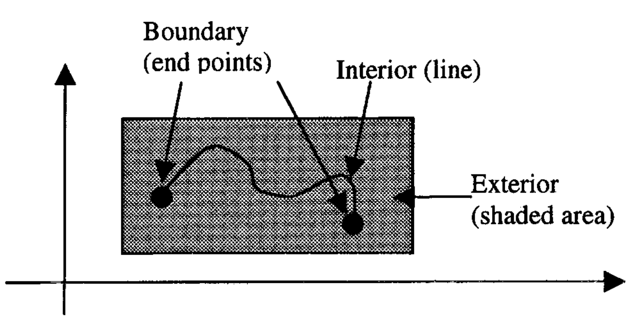
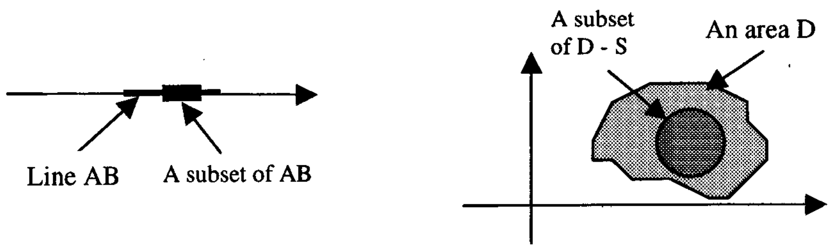


Figure 4. Topological paradox in IR^2—interior meets exterior if topological concepts in IR are simply adopted in IR^2.



(a) Line AB is an open set in *IR* (b) Area D is an open set in IR^2

Figure 5. The "open set" concept in 1-D and 2-D vector spaces.

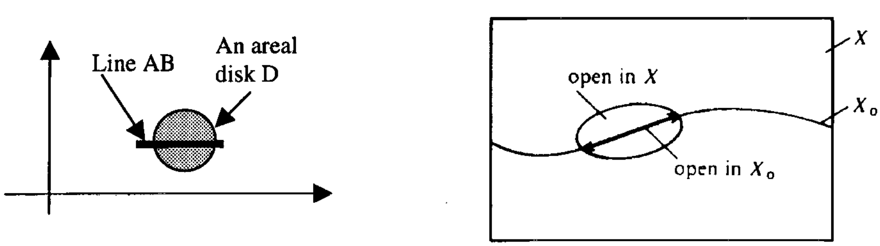
open neighborhood in IR^2) to be contained by *AB*, therefore *AB* is not an open set any more in this case [5], as illustrated in figure 6(a). Similarly, a curve feature in IR^2 is also not an open set, as illustrated in figure 6(b) [5].

3.4. Embedding low-dimensional entities into a higher-dimensional vector space

Figure 6 illustrates that an open set in *IR* is not an open set any more in a higher dimension, e.g., IR^2. It can be generalized that when a spatial entity of low dimension is embedded into a higher dimension, it is not an open set any more.

If a set is not an open set, it means that there are no such points in this set which have an "open neighborhood" contained by the set. This means that the set has an empty interior. For example, the line segment *AB* in IR^2 as shown in figure 6(a) has an empty interior. It means that a line feature in IR^2 has only a boundary and the boundary is the line itself but not the two end-points any more. Similarly, an area will have an empty interior in IR^3.

It is now clear that the paradox described in Section 3.2 is created by an implicit assumption that a line is still an open set in IR^2. Under such an assumption, the line has its two end-points as its boundary and the rest of the line segment as its interior. However, the line in IR^2 is not an "open set" any more, so that such concepts as closure, interior and



(a) Line AB is not an open set in IR^2 (b) The concept of "open" in IR^2 [6]

Figure 6. The "open set" concept in 1-D and 2-D vector spaces.

boundary (which are based upon the concept "open set") need to be re-examined. Indeed, as discussed previously, a line in IR^2 has an empty interior and the paradox disappears automatically.

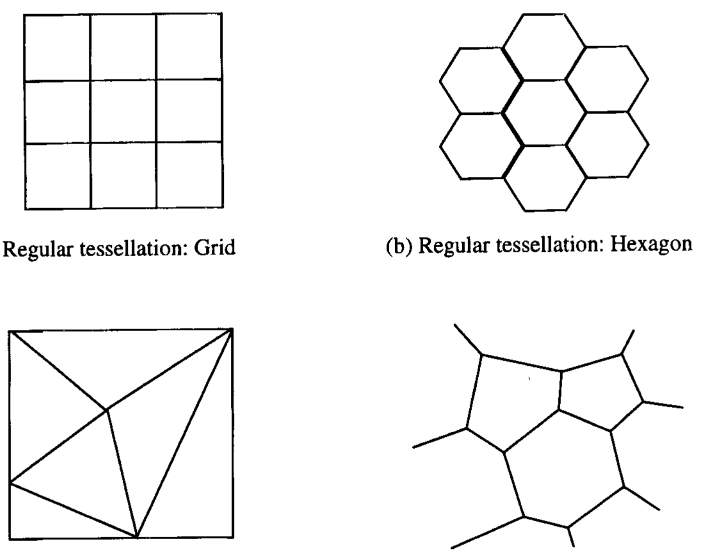
4. Topological issues in raster space

After the discussion of topological issues in IR^2, it is now pertinent to turn discussions to raster space. Space with a topological structure corresponding to IR^2 in raster space is denoted as Z^2 (Z^2 is the Cartesian product of Z and Z, where Z is the integer set). The topology in Z^2 was also termed as "digital topology" by [6].

4.1. The topological structure of raster space Z^2

The vector space is a continuous space. All the topological concepts are implicitly built upon the concept of connectedness. For example, it is commonly accepted that the two points in IR^2 are connected because the IR^2 is a connected space. However, in discrete space, this implicit assumption of connectedness no longer works.

A discrete space is a result of partitioning a connected space into small pieces which cover the whole space contiguously. This is also called space tessellation. Various approaches are possible, such as feature-primary and space-primary tessellation [7], and



(a) Regular tessellation: Grid

(b) Regular tessellation: Hexagon

(c) Irregular tessellation: TIN

(d) Irregular tessellation: Voronoi diagram

Figure 7. Some examples of space tessellation.

some examples of space tessellation are illustrated in figure 7. Grid-based tessellation is only one of them.

The tessellation based on squared-grids constitutes raster space Z^2 (i.e., the Cartesian product of Z and Z, where Z is the integer set). Each grid is viewed as an element, called ''spel'' (short for spatial element). All the spels in the plane can form a new set, which can be named as grid set G. The G can then be regarded as the hardware of the raster space and the transitive closure (δ) of the adjacency relation between the two spels in G. This system can be expressed as $\langle G, \delta \rangle$, where δ is the binary relations. This binary relation determines the connectedness between the spels in G.

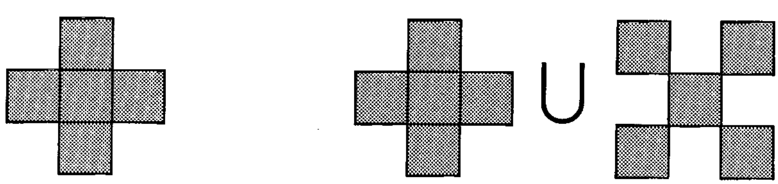
In 2-dimensional space, the adjacency between two neighboring spels can be defined in two different ways, i.e., 4-adjacency or 8-adjacency (see figure 8). With 4-adjacency, a spel is considered as being connected to neighboring spels along only four sides, i.e., up, down, left and right sides (figure 8a). However, in 8-adjacency, a spel is also considered as being connected to neighboring spels along four diagonal directions, i.e., upper/left, upper/right, lower/right and lower/left (figure 8b).

In 3-D space, there is a similar problem. In this case, it is 18-adjacency or 26-adjacency. More detailed discussion regarding this issue can be found in the paper by Kong and Rosenfeld [6].

In fact, raster topology $\langle G, \delta \rangle$ is normally referred to as ''digital topology''. This allows us to discuss topological problems related to spatial entities in Z^2, but not in R^2. The raster topology in R^2 is defined as $\langle R^2, \chi \rangle$, where χ is a family of unions of elements in G plus an empty set and R^2. With such definitions, in $\langle R^2, \chi \rangle$, the second axiom of vector topology stated in Section 3.1 becomes unnecessary.

4.2. *Basic topological components of a spatial entity in Z^2*

The basic topological components of a spatial entity (E) in raster space are still interior, boundary and exterior. The interior of E in Z^2, E^o, is the largest connected part of G contained in E and the closure of E. $\bar{E}$ is the smallest connected part of G containing E. The boundary as defined in vector space, ∂E, is the intersection between the closure of E and the closure of the complement of E, $\bar{E}^-$.



(a) 4-connectedness (b) 8-connectedness

Figure 8. Definitions of 4-connectedness (i.e., connected along four sides only) and 8-connectedness (connection along four sides and four diagonals).

The relationship between interior, closure and boundary is as follows:

$$E^o \cap \partial E = \Phi$$

$$E^o \cup \partial E = \bar{E}$$

$$\partial E = \bar{E} \cap \bar{E}^- .$$

The boundary of a spatial entity in raster space, as shown in figure 9(a), needs to be presented by a string of spels [6], which has an area. According to equation (1), most appropriate spels defining the boundary of the entity E are the two strings of spels shaded in figures 9(b) and 9(c). This boundary string is not difficult to obtain if the ''any pixel intersected'' method is used in rasterization, as shown in figure 9(e). However, in practice, the raster representation of E could be something like the E' shown in figure 9(d) if the

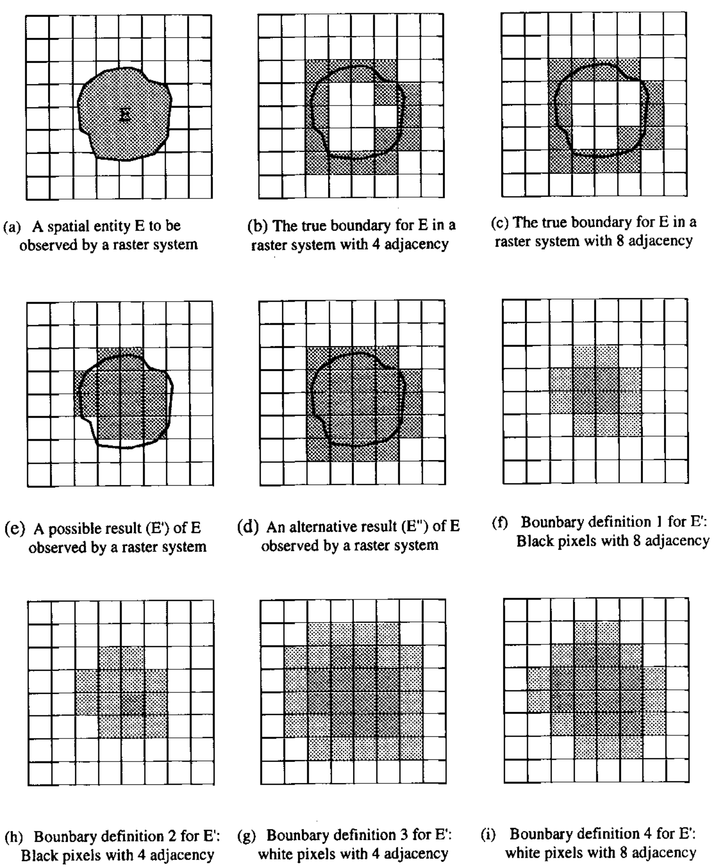


Figure 9. Boundary of an area entity in raster space.

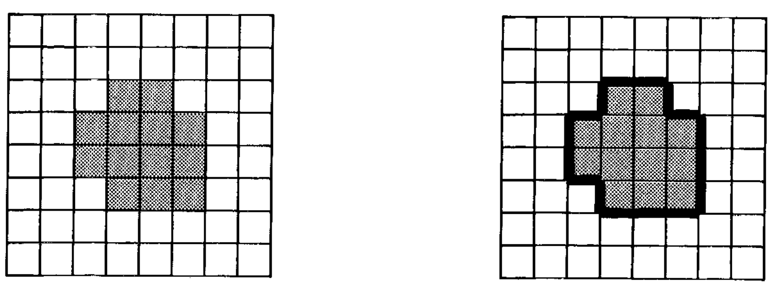
"dominant pixel" method is used in rasterization. This might also be closer to the case of imaging in raster format.

With E' as the raster representation of E, the boundary E' might be defined in a few alternative ways, as illustrated in figures 9(f) to 9(i). In the case where black spels in the outer ring of E' are used to form the boundary, the interior of E will become smaller than it should be. On the other hand, if the white spels neighboring the outer ring of E' are used to form the boundary, the interior of E will become larger than it should be.

These definitions of boundaries may cause some topological problems. In the case of "white boundary", the two disjoint spatial entities may share a common boundary. On the other hand, in the case of "black boundary", an areal entity may have an empty interior. Efforts have also been made to avoid such problems. For example, Egenhofer and Sharma [2] set a number of conditions in the treatment of raster entities. Indeed, in image processing literature, some researchers even suggest the use of a vector representation of the boundary for E', as illustrated in figure 10.

4.3. Topological paradox associated with definition of adjacency raster space Z^2

The adjacency definition is important not only in the computation of raster distance between two spels but also in topological analysis. [11] discussed a paradox related to the definition of adjacency, which is illustrated in figure 11. In this figure, there are four black spels and some white spels. One white spel is surrounded by the four black spels. If 8-adjacency is defined, the black spels are connected and should form a closed line; however, this black line cannot separate the central white spel from the rest. If 4-adjacency is defined, the black spels do separate the central white spel from the rest; however, these black spels are totally disconnected and thus no closed line has been formed by the black spels in this case. This is another interesting phenomenon but in raster space. It violates the so-called Jordan Curve theorem in vector space. This theorem is the basic topological property in vector space and it would be preferable to keep it in the Z^2 raster space. This being the case, a topological paradox has arisen (figure 11).



(a) Spatial entity E' observed by a raster system (b) Vector representation of boundary for E'

Figure 10. Spatial entity E' observed by a raster system and its vector representation of the boundary for E'. This boundary can be regarded as the result observed by Euclidean topology on R^2.

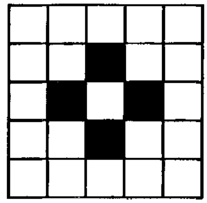


Figure 11. Topological paradox in raster space (modified from Maguire et al. [9]).

It has also been realized by Kong and Rosenfeld [6] that this problem could be solved if the white spels are defined as being 4-connected and black spels 8-connected, or vice versa. However, the question "why this paradox happens" still needs to be examined in more detail before any discussion on the topology in Z^2 raster space can be further discussed.

To explain this paradox more clearly, only four spels are used in figure 12, where point *P* is the intersection point of these four spels. According to the 8-adjacency definition, the black spels, i.e., spels 1 and 4, are connected. Similarly, spels 2 and 3 are also connected. As a consequence, two questions arise:

(a) Does P belong to the black line or the white line? and
(b) What makes spels 1 and 4 (or spels 2 and 3) connected?

These two questions are inter-related. In reality, when one considers spels 1 and 4 to be connected, one has already implicitly assumed that P belongs to the black line. On the other hand, when one considers spels 2 and 3 to be connected, one has already implicitly assumed that P belongs to the white spels. That is, the point P belongs to two different things. If the black spels represent spatial entities and the white spels represent the background, then point P belongs to both the background and the entity at the same time, thus having dual meanings. This of course leads to paradox—a kind of ambiguity.

To solve the problem, one must eliminate the dual meanings of point P. One should only allow P to belong to either the entity or the background but not both.

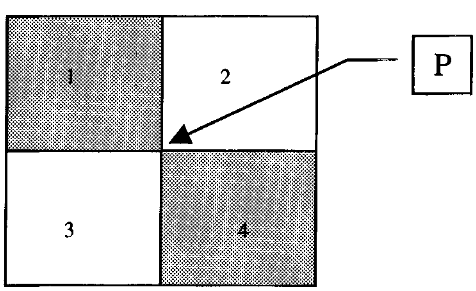


Figure 12. The ambiguity at point P, causing topological paradox.

5. Relationship between topologies on raster and vector spaces

After the discussions on the basic topological models for spatial entities in both vector space and raster space, the time has come to examine the relationship between these two topologies. As connectedness is the most fundamental concept in raster space, discussions will start from this concept.

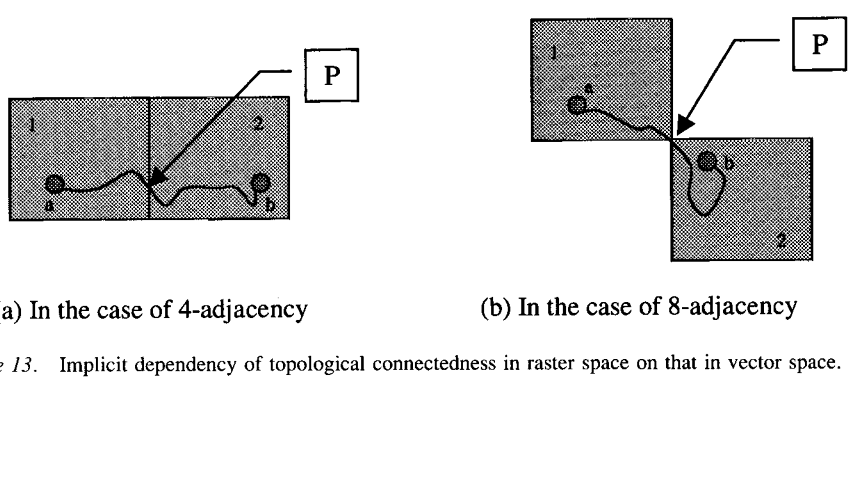
5.1. Dependency of raster topology on vector topology

The connectedness of raster space is based on the adjacency of two neighboring spels. The connectedness of two neighbouring spels in 4-connectedness and 8-connectedness is illustrated in figure 13. There is a common line (figure 13a) between the two spels in the case of 4-connectedness. On the other hand, in the case of 8-connectedness, the common part could be either a line (figure 13a), a point (figure 13b), or both. In other words, there is at least a point in common if the two spels are to be connected.

If an arbitrary (vector) point is selected from each spel, say ''a'' and ''b'', then the path from ''a'' to ''b'' intersects the common line at P. Points ''a'', ''b'' and P are points in vector space. Points ''a'' and P are connected in the left spel and points P and ''b'' are also connected in the right spel in vector space. As the connectedness is transitive, points ''a'' and ''b'' are therefore connected. As a result, any point in the left spel is connected to any point in the right spel. It means that the connectedness concept in vector space has been implicitly adopted when the connectedness concept in raster space is discussed.

5.2. Raster topology as a means for the study of vector topology

Suppose a subset in R^2 represents a spatial entity, it can be studied in vector space. It can also be studied in raster space. When an entity is studied in raster space, it is implied that



(a) In the case of 4-adjacency (b) In the case of 8-adjacency

Figure 13. Implicit dependency of topological connectedness in raster space on that in vector space.

there is a relationship between vector space and raster space. The observation of the entity has been transformed from vector space to raster space. In this transformation, some kind of invariability is retained or implicitly inherited, i.e., the connectedness. This connectedness invariant is the fundamental reason why the topological properties and relations of spatial entities can also be studied in raster space.

Indeed, it can be considered that raster topology is the quotient space of vector topology. Each topological relation of entities in raster space has its equivalence in vector space. Study of entities in raster (discrete) space could be more efficient than in vector space, as the expression of spatial entities in discrete space is more explicit than that in connected space.

6. Conclusions

In this paper, topology is regarded as a generalised system and the software of the system is used to approximate spatial entity. Topological models for multi-dimensional spatial entities in both vector space and raster space have been presented. The relationship between the topologic models in raster space and those in vector space is examined.

It has been noted that, in vector space, an open set in 1-D space is not an open set any more in 2-D and 3-D. Similarly, an open set in 2-D vector space is not an open set any more in 3-D vector space. As a result, fundamental topological concepts such as the boundary and interior of a spatial entity are no longer valid when a lower dimensional spatial entity is embedded in higher dimensional space. For example, in 2-D, a line has no interior and the line itself (not its two end-points) forms a boundary. Failure to recognise this fundamental topological property will lead to topological paradox.

The definition of the boundary of a raster spatial entity is more complicated. The raster boundary in Z^2 is quite different from that in R^2. The boundary of a raster spatial entity is also dependent on the definition of adjacency or connectedness of raster space. If connectedness is not carefully defined, topological paradox will be caused.

It has also been noted that, in raster space, the basic topological concept in vector space—connectedness—is implicitly inherited. This is why the topological properties of spatial entities can also be studied in raster space. Topological paradox will be caused if the ''connectedness'' of raster space is not carefully defined. Study of entities in raster (discrete) space could be more efficient than in vector space as the expression of spatial entities in discrete space is more explicit than that in connected space.

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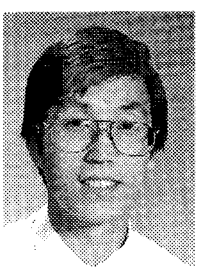

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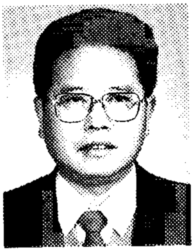

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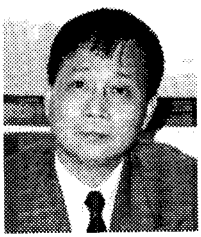

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