Robust Surface Matching for Automated Detection of Local Deformations Using Least-Median-of-Squares Estimator

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Abstract

Automated detection of the local deformation of a surface involves the detection of the differences between an original and a deformed digital surface model without the aid of control points. The process is normally automated by matching two digital surface models. This technique is desirable for many industrial applications.

With the existence of local deformation, conventional surface matching algorithms with least-squares conditions will fail because the estimated parameters are influenced by local deformation. As a result, some robust estimators can be applied to robustify surface matching algorithms. In addition to a re-evaluation of the performance of the M-estimator, two other robust estimators—the GM-estimator and the LMS-estimator (least median of squares)—have been explored in this study for the purpose of detecting local deformation. Test results show that the LMS-estimator is superior to both the M-estimator and the GM-estimator for detecting local deformation in three respects: (1) it is not sensitive to the location of local deformation; (2) the largest tolerable deformation percentage is improved to a level of almost 50 percent; and (3) when the deformation percentage is less than 40 percent, deformations of very small magnitude can be detected. It has also been found that the largest tolerable deformation percentage is related to the magnitude of the deformation.

Introduction and Background

In this digital era, almost everything is represented digitally (in databases), such as digital models of terrain surfaces, digital models of the human body, digital models of industrial products, digital models of engineering constructions, and so on. With these surface models, analyses can then be made, such as area and volume computation; designs and plans can be made, such as architectural design; information can be extracted, such as slope maps and contour maps; and so on.

This paper discusses a particular type of analysis of digital surface models—automated detection of local deformation or automated detection of the differences between two digital surface models. Such techniques are required in many industrial applications, such as monitoring surface deformation and soil erosion, product quality control, and object recognition.

Generally speaking, automated detection of the differences between two digital surface models is processed in two steps: i.e., automated matching of two surfaces and then automated detection of their differences. Here, "matching" means "registering." Therefore, surface matching means to register one digital surface model to the other. In this paper, for simplicity, the term "surface deformation" is used to refer to the differences between two surfaces.

After the matching is completed, the surface deformation can then be detected. However, the problem is not that simple. In reality, two surfaces cannot be precisely matched until the deformation has been identified and removed, while the deformation cannot be precisely detected (and then removed) until the two surfaces have been precisely matched. Therefore, surface matching must be performed either simultaneously, or in iteration, with the procedure of deformation detection.

From the literature on computer vision and close-range photogrammetry, it can be found that many surface matching algorithms have already been developed by researchers. These algorithms can be classified into two types (Zhang, 1994), i.e., the primitive-based approach and the area-based approach. In the primitive-based approach, some features are first extracted from both surfaces, correspondences between these features are then established through comparing their properties, and, finally, transformation parameters are computed using these correspondences. Features to be extracted could be the so-called critical points (e.g., points of large curvature) (Goldgof *et al.*, 1988), contours characterizing surface variation (Rodríguez and Aggarwal, 1990; Stein and Medioni, 1990), and surface patches acquired by segmenting the surface according to sign variation of curvature (Lee and Milios, 1990; Fan, 1990). Generally speaking, the primitive-based matching approach is capable of dealing with surfaces with large differences in orientation and is desirable for the purpose of object recognition (Zhang, 1994) due to the concise information used for matching, which is invariant to orientation differences. However, this approach has three main shortcomings, i.e., heavy computation and thus inefficiency and less accuracy, because the procedure of extracting features is highly sensitive to random errors, the abrupt variations of surfaces, and so on. On the other hand, area-based matching makes use of the overall features of the surface of concern, such as correlation and coherence between two surfaces, instead of individual features. Information regarding the surfaces, i.e., the coordinates of sample points in the case of digital surfaces, is used directly with little or no abstraction. Area-based matching is therefore able to produce more accurate matching due to the high redundancy and no undesirable loss of precision caused by pre-processing. The main defect of area-based matching is that some prior knowledge regarding the transformation is needed, because this

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approach to matching is essentially a heuristic search in the solution space for transformation parameters. Least squares is the technique most widely used in area-based matching (Rosenholm and Torlegård, 1988; Pilgrim, 1991; Besl and Mckay, 1992; Chen and Medioni, 1992; Zhang, 1994), although some other conditions are also used, such as the energy function (Szeliski, 1988). As a result of the wide use of the "least-squares" condition, area-based surface matching is simply called "least-squares matching" by many. One of the major, if not the most, significant advantages of least-squares matching is that it can be integrated with techniques for detecting surface differences.

In this paper, no attempt is made to extensively evaluate these algorithms. Instead, the aim is to robustify algorithms for matching surfaces with deformations, so that reliable surface matching can be achieved and thus deformation can be detected accurately. There are many types of deformation, as listed in Table 1. However, only local deformations will be investigated in this study. Because accuracy is the most important factor in the detection of deformations, an area-based matching algorithm using least-squares conditions will be selected for investigation in this study.

Several studies on the automated detection of local deformations have been carried out. Pilgrim (1991;1996) modifies the method suggested by Rosenholm and Torlegård (1988) for surface matching and then robustifies the methods using a maximum-likelihood estimator (M-estimator)(Huber, 1981). He reports that a surface with up to 25 percent local deformations (in terms of area size) can be detected. Karras and Petsa (1993) use the matching algorithm but with a data-snooping technique instead of the M-estimator. They do not indicate the capacity of their method, but they do indicate its problems by saying "detection of spatially localized deformations appears to be more complicated than detection of isolated outliers." It is rather clear that the capacities of existing methods are quite limited. It is therefore very natural and should be of interest to many researchers to explore other types of robust estimators for local deformation detection. Indeed, this is the main objective of this study.

In this paper, this introduction is followed by a theoretical discussion on how local deformations can be treated, what methodology can be used, and how conventional surface matching algorithms can be integrated with robust estimators. Then, principles and criteria involved in the design of experimental testing are discussed. An evaluation of existing robustified methods which make use of M-estimators is carried out. After that, some other robust estimators, particularly the least-median-of-squares estimator, are explored. These newly developed methods are evaluated and compared with existing methods.

Detection of Local Deformation: Theoretical Background

To make use of robust estimators for the detection of local deformation implies that local deformations are treated as outliers and robust estimators are estimation techniques which are less sensitive to the existence of outliers. It is therefore necessary to examine the theoretical basis of this approach before detailed techniques can be presented.

Local Deformations as Outliers

Supposing that two digital surface models can be precisely matched by taking into consideration only the non-deformed portions. Then the residuals of all the surface points can be divided into two categories: those coming from the non-deformed parts (denoted by R_I) and those coming from the deformed parts (denoted by R_D). The elements in R_I can be assumed to obey the same normal distribution $N(0, \sigma)$. The mean of elements in R_D would usually differ significantly from zero and the standard deviation would also be large to some extent. When R_I is mixed with R_D, the elements in R_D could therefore be considered as outliers (or gross errors) in R_I, if the percentage of deformed areas is within a certain limitation. This brings to us the idea that local deformations could be detected as outliers in observations of the matching process.

From the literature, it can be found that there are two major approaches to dealing with outliers, i.e., outlier detection and robust estimation. In the former, one tries first of all to detect (and remove) outliers, and then to perform estimation with the "clean" observations. Some statistics based on residuals (usually resulting from a least-squares adjustment) are designed to measure the "outlyingness" of the observations. The most popular statistics are the normalized residuals developed by Baarda (1968) and the studentized residual proposed by Pope (1976). The statistical testing procedure using either of these two statistics is known as data snooping. In data snooping, given a desired level of significance, a threshold is determined and then any observation with a normalized residual exceeding this threshold is detected as an outlier. The lower bound of gross error detectable with a given probability can also be estimated (Förstner, 1986; Li, 1988). However, data snooping is only applicable under the assumption that no more than one outlier is present in a data set. As concluded by Karras and Petsa, (1993), "detection of spatially localized deformations appears to be more complicated than detection of isolated outliers."

Although some methods for the detection of multiple outliers, such as iterative data snooping, have also been proposed (e.g., Kok, 1984; Gao *et al.*, 1992), these methods are still not practical for the detection of surface deformation, because the number of points on the deformed parts of a surface could be large. As a result, attention has been paid to robust estimation.

Robust methods (Huber, 1981; Hampel *et al.*, 1986) employ estimators that are relatively insensitive to the presence of outliers, so as to produce reliable estimates. As a result, outliers can be identified. From the literature, it can be found that the so-called M-estimators (maximum-likelihood type estimator) have been used for the detection of local deformations (e.g., Pilgrim, 1991; Pilgrim, 1996). However, other types of robust estimators have not yet been studied for this purpose. Therefore, in this study, both theoretical and experimental evaluations will be carried out of the performance of various robust estimators for matching surfaces with local deformations.

Surface Matching Algorithm and Its Robustification

As has been pointed out previously, one of the major advantages of least-squares matching is that it can be integrated with techniques for detecting surface differences. Due to this characteristic, the least-squares approach was selected for this study. In particular, the Iterative Closest Point (ICP) algorithm proposed by Zhang (1994) and the Least-Height Difference (LHD) algorithm proposed by Rosenholm and Torlegård (1988) were applied in this study. It was found that the latter performed slightly better in our tests. Therefore, only the study of the latter will be reported.

To facilitate the understanding of the rest of the paper, the matching algorithm is briefly introduced here. The objective of matching is to find the transformation (**T**) between two surfaces, which consists of a rotation (**R**) and a translation (**t**), such that the "distance" between the two surfaces is minimized. For the LHD algorithm, the height difference between the two corresponding points is used as the measure of distance. Let x and x' be corresponding points on the two surfaces S and S'. If the

Table 1. Classification of Surface Deformation

Criteria	Type of Deformation
Magnitude of deformation	Small, medium, large, etc.
Size of deformed area	Local, global
Characteristics of deformation	Systematic, non-systematic, random

two surfaces, S and S′, are registered by a transformation **T**, then the height (H'_x) of point x', after applying **T**, will be the same as the height (H_x) of point x except for random errors, i.e., the following criterion holds:

$$F(\mathbf{R}, \mathbf{t}) = \sum_{j=1}^{n} p_j r_j^2 = \sum_{j=1}^{n} p_j (Hx_j - H'_{\mathbf{R}'x_j+\mathbf{t}})^2 = \min. \quad (1)$$

The weight p_j, which takes value 1 or 0, is introduced to deal with the problem that two surfaces don't cover exactly the same range.

However, we are not clear about the pairing relationship between the two sets of points, because there are no control points. An approximation is used to pair points off. Given that the orientation difference between two surfaces is small, two points on their respective surfaces that have the same location on the (x, y) plane are temporarily paired, and the surface is assumed to be smooth to some extent. By doing pairing and minimization iteratively, every iteration will bring the two surfaces closer so that they finally match. We would point out only that both the ICP and LHD algorithms are very precise and have a reasonably large pull-in range, and thus are very suitable for free-form surface matching. For a detailed discussion, please refer to the above-mentioned references.

The mathematical expression of the general form of robust estimators can be written as follows:

$$g(\rho(r_i)) = \min. \quad (2)$$

In the case of the LS-estimator (least-squares estimator), $\rho(r_i) = r_i^2$, and $g = \Sigma$ (i.e., summation). With a robust estimator, the condition of a surface-matching algorithm becomes

$$F(\mathbf{R}, \mathbf{t}) = g(\rho(r_i)) = \min. \quad (3)$$

Design of Experimental Tests: Methodology

In this study, the LHD algorithm will be robustified with various robust estimators, and the performance of these improved algorithms will then be evaluated with experimental tests.

Benchmarks for Performance Tests

In the evaluation of a robust estimator, breakdown point and relative efficiency are the two criteria to be considered.

The breakdown point of an estimator is the smallest fraction of outlier contamination that can cause the estimates to be arbitrarily biased (Rousseeuw and Leroy, 1987). For example, the breakdown point of the least-squares estimator is 0 (zero), because a single outlier can (but does not necessarily) make the result invalid. The breakdown point offers a referential value of the largest tolerable ratio (in terms of percentage) between the deformed areas and the total surface area within which local deformation can still be treated as an outlier. Such a value is referred to as the largest percentage of detectable deformation. The relative efficiency of an estimator is defined as the ratio between the lowest achievable variance (the Cramer-Rao bound) for the estimated parameters and the actual variance provided by the estimator (Meer *et al.*, 1991; Huang, 1990). The higher the relative efficiency of the estimator, the more precise the estimated parameters, and thus the smaller the magnitude of local deformation that can be detected. For example, the LS-estimator (least-squares estimator) has a relative efficiency of 1 in the presence of normally distributed errors, and the median as a one-dimensional location estimator has a relative efficiency of 0.637.

Therefore, the following two criteria, i.e.,

- the smallest magnitude of deformation and
- largest percentage of deformed areas,

should be used as benchmarks for the evaluation of the robust estimator for the detection of local deformation. There is a trade-off between these two factors (Kumar and Hanson, 1994; Huang, 1990). It is the experimental investigation into this trade-off that constitutes an important part of this study.

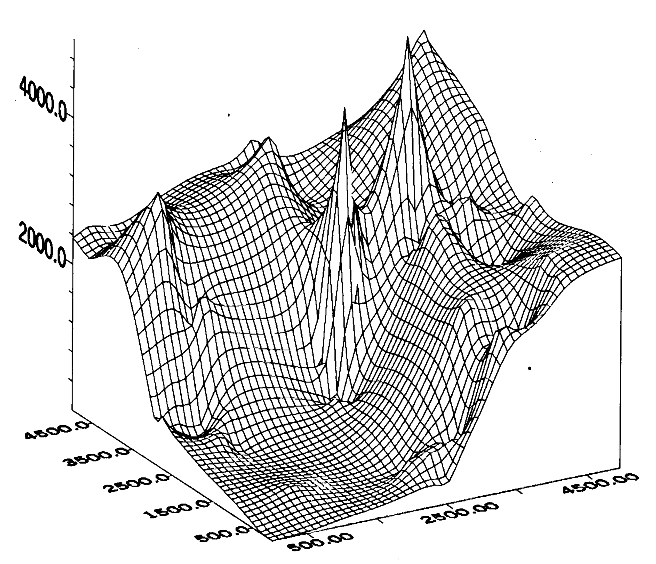

Figure 1. A perspective view of the surface used for testing.

Test Data Sets

In this test, a surface as shown in Figure 1 is used for exhaustive testing of the robustified surface matching algorithm. The surface consists of a 50 by 50 grid. The grid interval is assumed to be equal to 100 units. The difference between the maximum and minimum height is about 5000 units. To have a more clear view of the surface, the scale in the Z direction is reduced to produce the diagram shown in Figure 2. This surface can be considered as surface S. The corresponding deformed surface S′ (called the mate surface in this paper) is generated by adding a local deformation and random errors to the heights value of S. The mate surface is then rotated and translated by a parameter vector **Θ**. The robustified (i.e., with robust estimator) matching algorithms are used to compute the elements of parameter vector **Θ**.

Independent Variables of the Experiments

In the experimental test, the effect of the following three variables on the performance of surface matching will be investigated:

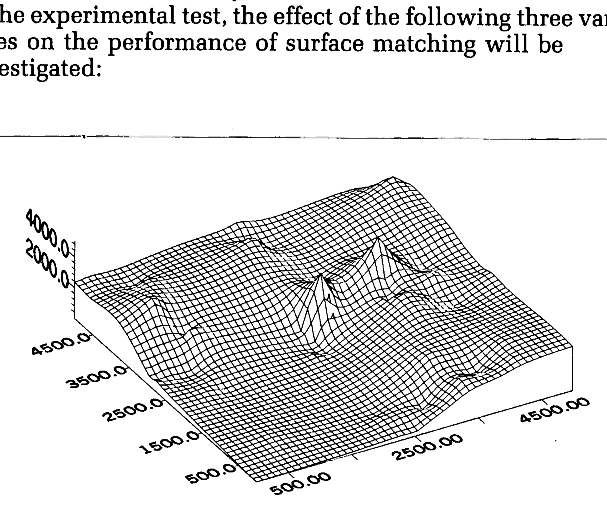

Figure 2. A Z-scale reduced view of the surface in Figure 1.

- the percentage of the deformed areas,
- the magnitude of the deformation, and
- the location of the deformations.

The simulated deformation is assumed to take the form of an $N(c, \sigma)$ distribution. The simulated random errors take the form $N(0, \sigma)$. The shape of the deformation is very flat. With varying c, the magnitude of the local deformation can be controlled. The simulated deformation is always gathered because it is well accepted that gathered deformation is harder to deal with than scattered deformation.

It can be suspected that the performance of some algorithms may be affected by the location of the deformations. In this study, deformations have been added to all of the nine positions, i.e., upper/left, upper, upper/right, left, middle, right, lower/left, lower, and lower/right. This doesn't mean that the largest percentage of deformation is 1/9. Instead, the percentage can be increased to any amount and is one of the most important variables to be considered.

M-Estimators for Local Deformation Detection: An Evaluation

Although the use of the M-estimator for detecting surface deformation has been reported by other researchers, the authors feel the necessity to re-evaluate it in terms of the above proposed benchmarks, which is the basis for comparing different robust matching methods. The M-estimator is a type of estimator that minimizes the sum of a symmetric and positive-definite function of residuals, $\rho(r_i)$ as shown in Equation 2, which has a unique minimum at $r_i = 0$. Mathematically, the general form of the M-estimator can be written as follows:

$$\sum_{i=1}^{n} \rho(r_i) = \min. \tag{4}$$

Let $\psi(r_i) = \frac{d}{dr}(\rho(r_i))$; then the following equation holds:

$$\sum_{i=1}^{n} \psi(r_i) \times \frac{\partial r_i}{\partial \theta_j} = 0 \tag{5}$$

where θ_j represents unknowns to be estimated.

In order for an estimator to be robust, the $\psi(r_i)$ function of an M-estimator must be bounded. For the estimator to be of high efficiency, $\psi(r_i)$ should be linear in the range near where $r_i = 0$. With consideration of these two elements, a number of $\psi(r_i)$ functions have been constructed, leading to different M-estimators. One particular estimator with desirable properties of robustness is Turkey's M-estimator, of which the $\psi(\ )$ function is as follows:

$$\psi(u) = \begin{cases} u \cdot \left(1 - \frac{u^2}{c^2}\right)^2 & |u| \le c \\ 0 & |u| > c \end{cases} \tag{6}$$

where $u_i = r_i/\sigma$, c is a tuning constant, and σ is the standard deviation of observations. For M-estimators, it is important to use a robust estimate of σ. In this study, σ is computed as

$$\hat{\sigma} = 1.4826\ med\ |r_i - med\ (r_i)| \tag{7}$$

which is fairly robust, with a breakdown point of 0.5 (Rousseeuw and Leroy, 1987). The constant 1.4826 is used to achieve consistency with the normal error distribution.

In this study, this M-estimator will be picked for comparative analysis with other types of estimators, such as the integral of the M-estimator and data snooping, and the LMS (least-median-of-squares) estimator.

In computation, M-estimators are usually converted to iterations of reweighted least squares, where weights are determined using the function $w(u) = \psi(u)/u$. That is,

$$\psi(u) = \frac{\psi(u)}{u} \cdot u = w(u) \cdot u. \tag{8}$$

The weights are updated during the iterative process according to the updated residuals and the estimate of the standard deviation of observations.

To incorporate the robust estimator into a surface-matching algorithm, the only change required is to replace the weight p_j in Equation 1 by $p_j \times w(u_j)$, i.e.,

$$F(\mathbf{R}, \mathbf{t}) = \sum_{j=1}^{n} p_j \times w(u_j) \times r_j^2 = \min. \tag{9}$$

Theoretically speaking, M-estimators have breakdown points less than $1/(p + 1)$, where p is the number of parameters to be estimated (Meer *et al.*, 1991). (Rigorously, M-estimators have breakdown points of 0 in the presence of high leverage outliers. However, it is normally assumed that no such outliers exist in the surface-matching process.) In the case of surface matching, p is 6 and the breakdown point is consequently less than 1/7 (0.143 or 14.3 percent). This is the theoretical upper bound. As another example, the median as a one-dimensional location estimator has a breakdown point of 0.5—the highest achievable value. Most M-estimators have a relative efficiency of above 0.9.

Several tests have been carried out on the surface-matching algorithm robustified with an M-estimator (LHD-M). The first is to test the accuracy of the matching. In this test, the mate surface model is generated by adding to the original digital surface model a random error of $N(0, 20)$ distribution and a local deformation of $N(200, 20)$ distribution. The percentage of the deformed area is 9 percent and the local deformation lies at the upper right corner (in the *XY* plane) of the digital surface. The results are listed in Table 2. It is clear that the estimates obtained by the M-estimators are much better than those by the LS-estimators.

The second test is on the largest tolerable percentage of deformed areas and smallest magnitude of formations. In this case, the percentage of deformation is increased from 4 percent to 25 percent, and the magnitude of deformation is increased from 3.5σ to 100σ. The results are shown in Table 3. It is clear that the largest tolerable percentage of deformed area is 16 percent and the smallest detectable magnitude of deformation is 3.5σ. The value of 16 percent is very close to its theoretical limitation.

Testing on the position of deformation has also been carried out. It was found that, generally, the further the deformation is located from the surface center, the smaller the largest tolerable percentage of deformed area. When the deformation is located in the middle of the area, a percentage of deformation larger than 16 percent can be detected. Indeed, in this case of this test, a value of 30 percent is obtained. This may explain why Pilgrim (1991; 1996) claims that the percentage is up to 25 percent. The results given in Table 3 are obtained when the local deformation is located in the upper/right corner. Tests on other de-centered deformations showed comparable results in terms of the largest tolerable percentage of deformed areas and smallest magnitude of formations, although it was not strictly symmetrical.

Least-Median-of-Squares and other Robust Estimators for Local Deformation Detection: An Exploration

From the testing results shown in the previous section, it is clear that the applicability of M-estimators in local deformation detection is very limited because the tolerable percentage

TABLE 2. THE ACCURACY OF THE LHD-M-ESTIMATOR ALGORITHM

		ω	φ	κ	t_x	t_y	t_z
True Value		00 00 00	00 00 00	00 00 00	0.00	0.00	0.00
LS-Estimator	No random error	00 14 46	00 11 32	−00 02 18	3.76	−4.62	−9.30
	With random error	00 14 05	00 12 28	−00 04 01	−3.29	−1.67	−7.83
M-Estimator	No random error	00 00 00	00 00 00	00 00 00	0.00	0.00	0.00
	With random error	−00 02 21	00 01 48	−00 00 43	−0.03	0.87	−0.71

NB: Percentage = 9%, deformation = N(200,20), random error = N(0,20), position = upper/right.

TABLE 3. EFFICIENCY AND BREAKDOWN POINTS OF M-ESTIMATORS

	4%	9%	16%	20%	25%
3.5σ	Success	Success	Occasional failure	Failure	Failure
4.0σ	Success	Success	Occasional failure	Failure	Failure
4.5σ	Success	Success	Occasional failure	Failure	Failure
5.0σ	Success	Success	Occasional failure	Failure	Failure
10.0σ	Success	Success	Success	Failure	Failure
50.0σ	Success	Success	Success	Failure	Failure
100.0σ	Success	Success	Success	Failure	Failure

NB: The results are obtained when the local deformation is located in the upper/right corner

of local deformation is only up to 16 percent. Therefore, it is quite natural to explore the feasibility of other robust estimators for local deformation detection, instead of trying to develop improved M-estimators as done by Pilgrim (1991; 1996). In this study, two other types of robust estimators are explored, and the results will be discussed in this section.

Integration of the M-Estimator and Data Snooping Technique

The first natural idea is to improve the performance of the M-estimator by integration with data snooping techniques, i.e., to take advantage of both techniques. In data snooping, one looks not for large residuals in an absolute sense but for residuals that are large in comparison to their own standard deviations (Schwarz and Kok, 1993). In other words, the redundancy numbers of individual observations should be taken into account. As a consequence, the following expression is used to compute the u_i in Equation 6:

$$u_i = \frac{r_i}{\sigma_{r_j}} \tag{10}$$

which is Baarda's statistic. The resultant estimator belongs to the category of GM-estimators (generalized M-estimators) (Rousseeuw and Leroy, 1987) which hope to bound the influence of leverage points. This estimator is here called the M-D-estimator because it can also be regarded as an integration of the M-estimator with a data snooping technique.

From a different point of view, the M-D-estimator is similar to iterative data snooping (Kok, 1984), while they differ in two aspects. First, with the M-D-estimator, more than one observation can be identified as a potential outlier after each iteration, while only one observation can be identified in the case of the iterative data snooping method. This is necessary for our application, because the number of points with deformation can be very large. Second, with the M-D-estimator, potential outliers are not excluded permanently from all successive iterations but; in the next iteration only, are given less weight. Experiments on the simple regression problem by other researchers (e.g., Rousseeuw and Leroy, 1987) showed that GM-estimators tolerate more outliers than do M-estimators. Testing of this M-D-estimator will be reported later.

Least-Median-of-Squares Estimator (LMS-Estimator)

As one can imagine, the performance of M-D-estimators is better than that of the M-estimator, but the improvement must be very limited. Therefore, a natural line of thinking is to explore a completely different type of robust estimator. From the literature, it can be found that the LMS (least-median-of-squares) estimator developed by Rousseeuw and Leroy (1987) has some good properties and is worthy of exploration. The mathematical expression can be written as follows:

$$\operatorname*{Med}_{i=1}^{n} r_i^2 = \min. \tag{11}$$

Comparing with Equation 2, it can be seen that $\rho(r_i) = r_i^2$ and $g = Med$. That is, the estimates must yield the smallest value for the median of squared residuals computed for all observations. In the case of robust surface matching, it means that we look for estimates of the transformation parameters that produce the median of the squares of height difference residuals smaller than any other estimates.

Theoretically speaking, the LMS-estimator has a breakdown point of 0.5, the highest possible value, which is desirable for an application for the detection of local surface deformation. The high breakdown point of the LMS-estimator is a result of the fact that the estimates need only to fit well for half of the observations, and the other half completely ignored. On the other hand, its relative efficiency is abnormally low because few observations contribute to estimates, as will be seen later. Actually, the LMS-estimator converges at a rate of $n^{-1/3}$ and is not asymptotically normal. To compensate for the low efficiency, it is recommended that an M-estimation or a one-step weighted least-squares solution be performed, taking the result of LMS estimation as the initial state (Rousseeuw and Leroy, 1987).

The LMS problem given by Equation 11 cannot be analytically resolved. A random sampling procedure is implemented to resolve the LMS problem, as recommended by Rousseeuw and Leroy (1987). The procedure progresses by repeatedly drawing sub-samples of (e.g., seven) points of size p from the mate surface. From such a sub-sample, we do an ordinary LHD matching, i.e., the sub-sample of points on the mate surface is matched to the original surface using the LHD matching algorithm. The resultant estimate of transformation parameters is denoted as Θ_i (trial estimate). For each Θ_i, we apply the transformation to the whole mate surface and determine the height difference residual for every point on the mate surface. The objective function with respect to all matchable points is then determined, i.e., the value $med\ r_i^2$ is calculated. Finally, the trial estimate with a minimal value of $med\ r_i^2$ is taken as the LMS estimate.

Rigorously speaking, a trial should be made for each of the possible sub-samples. Given the number of sample points of the mate surface, n, and the size of the sub-sample, p, the total number of sub-samples is C_n^p. However, the required computation could be prohibitively complex, because n is usually as large as from thousands to millions. This is why the random sampling technique is introduced. The basic idea is that one performs a certain number (m) of random selections of size p, such that the probability is almost 1 that at least one of the m sub-samples contains no outliers. In our case, it is the probability that at least one of the m sub-samples of points on the mate surface

Table 4. The Accuracy of Surface Matching by Different Versions of Robustified LHD Algorithms

	ω	φ	κ	t_x	t_y	t_z
True values	5°00′00″	5°00′00″	5°00′00″	500.0	500.0	500.0
LHD + LS-estimator	4°43′51″	4°46′17″	5°08′32″	491.6	493.8	507.7
LHD + M-estimator	5°01′36″	4°57′40″	5°02′18″	498.4	503.9	500.9
LHD + M-D-estimator	5°01′43″	4°58′02″	5°02′10″	497.7	503.1	500.7
LHD + LMS-estimator	5°01′44″	4°57′51″	5°01′39″	498.2	503.3	500.6

Random error ~ N(0,20), *deformation* ~ N(200,20), percentage ε = 9%, location ~ upper/left

belongs to the non-deformed area. The expression of this probability is as follows:

$$P(\varepsilon) = 1 - (1 - (1 - \varepsilon)^p)^m \quad (12)$$

where ε is the contamination fraction (percentage of deformed areas) and takes a value of 0.5 at most.

Given that ε is fixed, say 0.5, it's easy to find that m increases sharply with p to keep $P(\varepsilon)$ close to 1. A large m means a large number of trial matchings, i.e., a large amount of computation. Therefore, p should be set as small as possible in order to make m small. For the LHD algorithm, p must not be less than 6, because one pair of corresponding points gives only one observation equation, and there are six parameters to be determined. However, for a small p, the transformation difference has to be very small for the LHD to succeed, which relies on the comparability between the original surface and the mate surface. We have found from experiments that LHD matching using all matchable points made the transformation difference very small, although the parameters were significantly affected by the deformation (see Table 4 for an example). Therefore, in our implementation, an LHD matching using all matchable points is always performed to bring the two surfaces "close" to each other before the LMS estimator is applied. In addition, an M-estimator is finally applied to improve the precision of the LMS estimates, which are the result of a matching process using only a very few (say seven) sample points and therefore cannot be precise. In most of the experimental tests conducted in this study, the size of the sub-sample is set to be seven, and the number of sub-samples is set to be 600. Therefore, in the case of $\varepsilon = 0.5$, the above probability is approximately 0.991 (99.1 percent) and, in the case where ε is less than 0.5, the probability will be closer to 1. However, experiments show that when ε is greater than 0.4, 600 sub-samples are not enough for the LMS estimator to succeed in our deformation detection application. The number of sub-samples has to be much larger in these cases, as will be explained later.

Experimental Evaluation on the Performance of the M-D-Estimator and the LMS-Estimator

As for the evaluation on the performance of M-estimators, random errors and local deformations are added to the surface described in the section on Test Data Sets to obtain a deformed mate surface. Furthermore, rotations and translations are also applied. The testing results are listed in Tables 4 and 5.

Table 5. Trade-Off Between Smallest Detectable Magnitude (Relative Efficiency) and Percentage of Deformed Areas (Break-Down Point) for Various Versions of Robustified LHD Algorithms

Percentage	9%	16%	20%	22%	25%	36%	42%	46%	49%
LHD_m	3.5σ	4σ							
LHD_m-d	3σ	3.5σ	4σ	5σ					
LHD-lms	3σ	3.5σ	3.5σ	3.5σ	4σ	7σ	10σ	15σ	25σ

Table 4 shows the accuracy of surface matching algorithms robustified by various robust estimators. It is clear that all of them perform well. By contrast, the results obtained by the conventional LS-estimator are very poor. Table 5 shows the results of the smallest detectable magnitude of deformations by different algorithms with varying percentage of deformed areas. The results show that the LHD+M-D algorithm is capable of detecting local deformation with up to 22 percent of deformed areas, while the LHD+LMS algorithm does not break down even if the percentage of deformation is up to nearly 50 percent, as long as the magnitude of deformation is large enough.

Tests have also been carried out to make a comparative analysis of the performance of these robustified matching algorithms by varying the magnitude, percentage, and position of local deformation. It can be noted that these algorithms behave quite differently. The deformation percentage plays a dominant factor. In most cases, if the deformation percentage is over 16 percent, the LHD+M algorithm fails. The LHD+M-D algorithm is capable of coping with a deformation percentage as large as 22 percent. The LHD+LMS algorithm still performs reasonably well even if the deformation percentage is as large as 40 percent. Figure 3 shows the different performance of the three robust matching algorithms. In this case, the simulated deformation, located at the upper/left corner, occupies 20 percent of the whole area and its magnitude is 5σ. The upper three plots in Figure 4 show the orthographic graph of height differences after applying LHD+M, LHD+M-D, and LHD+LMS matching respectively from left to right. The grey level of the grid indicates the magnitude of the height difference at the specific point. The darker it is, the more the height difference deviates from 0. As can be seen, for both the LHD+M-D and the LHD-LMS algorithms, the height differences at all points clearly form two groups, one light and one dark, and we can also see the steep slopes at the boundary where the two groups meet. By contrast, in the LHD-M case, there are still two groups but the grey difference is much less sharp. The lower three plots show the detected points constituting deformation using a simple 3σ criteria. As can be seen, the LHD+M-D and LHD+LMS algorithms detected the deformation area accurately even at the deformation boundary. Although there are some erroneous detecting decisions, these are isolated noises and can easily be filtered out by applying some simple smoothing filters. However, the LHD+M algorithm did not give a desirable result. Figure 5 shows a case in which the LHD+M-D algorithm failed while the LHD+LMS algorithm succeeded. Figure 6 shows the power of the LHD-LMS algorithm in dealing with deformation that occupies a large area but is of small magnitude.

The results shown in Tables 4 and 5 are obtained when the local deformation is located at the upper/right corner. However, both the M-estimator and the M-D-estimator are sensitive to the position of local deformation. The farther the location of local deformation from the center (in the *XY* plane) of the digital surface model, the smaller the percentage of the deformed areas that can be detected. This can be understood by noting that observations far from their geometric center therefore have lower redundancy numbers and thus contribute more to estimates than those near the center. The largest detectable deformation percentages for the LHD+M and LHD+M-D algorithms are both about 32 percent if the local deformation is located at the center of the surface. The LMS-estimator is not sensitive to the position of local deformation at all because it does not favor leverage observations (observation of abnormally low redundancy numbers) (see Rousseeuw and Leroy, 1987).

Discussions and Analysis

The experience gained from the intensive experimental test reveals that the theoretical limitation of tolerable deformation percentage is applicable although the limit is exceeded in some cases where the deformation is located in the middle of the sur-

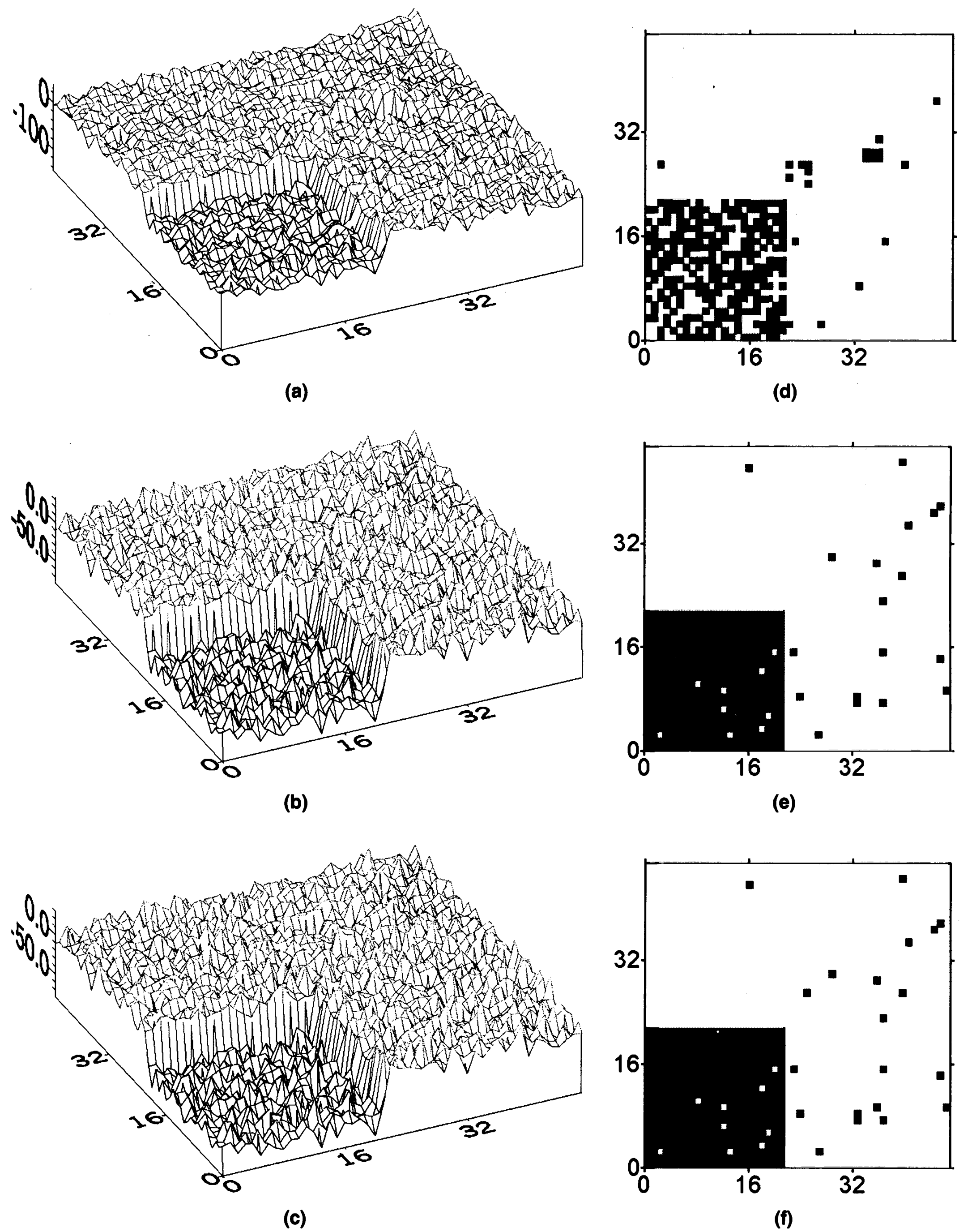

Figure 3. A comparison of LHD+M, LHD+M-D, and LHD+LMS algorithms (the deformation is located at the lower-left corner, occupies 20 percent of the entire area, and is of magnitude 5σ). (a) Height differences after applying the LHD+M algorithm. (b) Height differences after applying the LHD+M-D algorithm. (c) Height differences after applying the LHD+LMS algorithm. (d) Deformation area detected by the LHD+M algorithm. (e) Deformation area detected by the LHD+M-D algorithm. (f) Deformation area detected by the LHD+LMS algorithm.

face area. When the deformation percentages are within the tolerable range, both the LHD+M and the LHD+M-D algorithms are able to deal with local deformations of very small magnitude, i.e., as small as 3.0σ or 3.5σ. This is attributed to the high relative efficiency of the M-estimators, the extremely high redundancy, and the good distribution of observations. At a given deformation percentage, the LHD+M-D algorithm is capable of detecting local deformation with a smaller magnitude than that by the LHD+M algorithm. This is due to the employment of Baarda's statistic, which approximately takes into account the spatial positions of individual observations.

It can also be noted that the smallest detectable magnitude of deformation increases with the deformation percentage. This trend for the LHD+LMS algorithm is plotted in Figure 6.

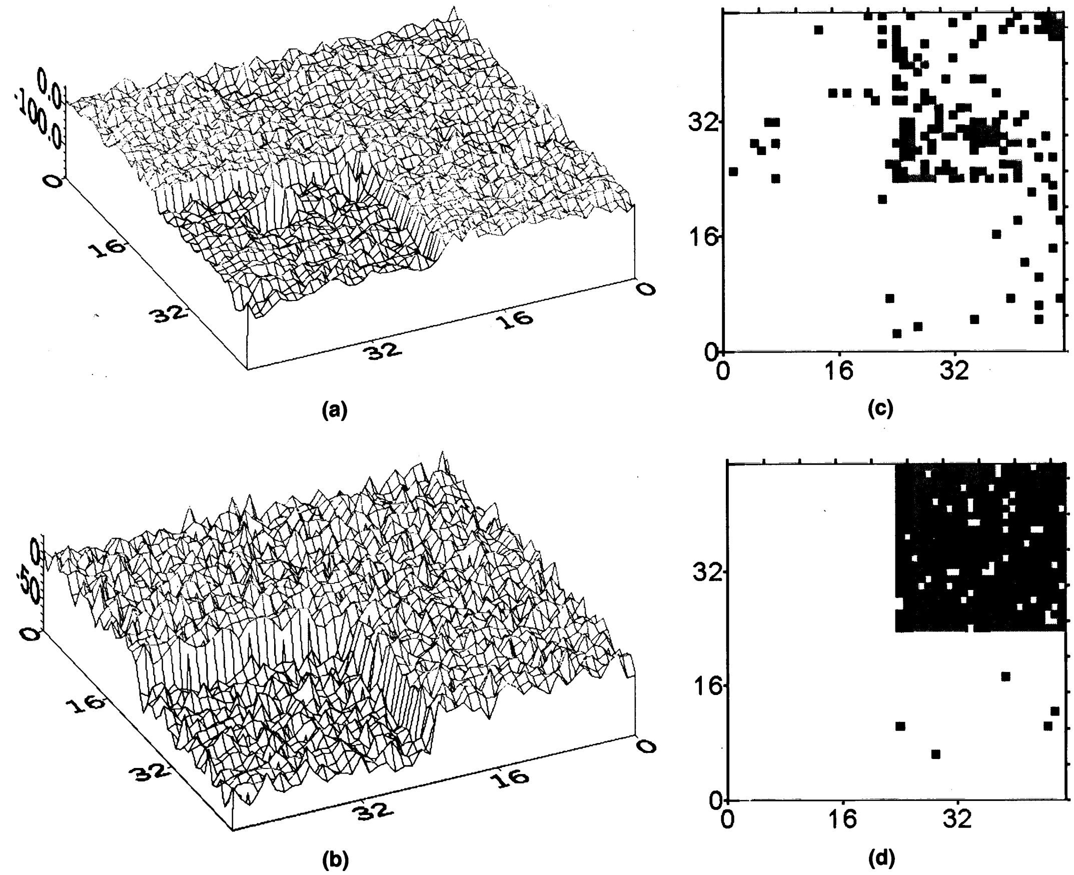

Figure 4. A comparison of the LHD+M-D and LHD+LMS algorithms (the deformation is located at the upper-right corner, occupies 25 percent of the entire area, and is of magnitude 5σ). (a) Height differences after applying the LHD+M-D algorithm. (b) Height differences after applying the LHD+LMS algorithm. (c) Deformation area detected by the LHD+M-D algorithm. (d) Deformation area detected by the LHD+LMS algorithm.

When the deformation percentage is less than 36 percent, the LHD+LMS algorithm is quite well behaved. However, as the deformation percentage goes over 36 percent, the smallest detectable magnitude grows acutely. On the one hand, one might say that the LMS-estimator fails due to its low efficiency. It is found that the value of the median of squares produced by the LMS-estimator implemented with a random sampling procedure is not smaller than that produced by the conventional LS-estimator if the percentage of deformation is high and the magnitude of the deformation is small. On the other hand, the failure of the LMS estimator is caused by the distribution pattern of local deformation. Although the LMS estimator is not sensitive to the location of deformation, the pattern of geometric distribution of local deformation does matter. When the area percentage of deformation is large (near 50 percent) and the deformation is gathered in one area, the geometric distribution of the "clean" sub-sample may be too poor to produce a precise estimation. We also find that, when the deformation percentage is above 40 percent, 600 sub-samples are not enough for the LMS estimator to succeed. In such cases, 3000 or even more are needed.

Conclusions

In this study, an investigation into the automated detection of surface deformation was conducted using automated surface matching techniques. The emphasis was on the detection of local deformations; therefore, no intensive investigation into the performance of surface matching itself was carried out. It was then reasoned that local deformation can be treated as an outlier, and robust estimators can be employed to robustify the surface matching algorithms. The performance of such estimators can be evaluated against a set of benchmarks, mainly the tolerable percentage of deformed areas and the smallest detectable magnitude of deformation.

From the literature, it was found that M-estimators have been employed by researchers for the purpose of detecting local deformation. Some researchers concluded that M-estimators are capable of detecting local deformation with over 25 percent (Pilgrim, 1991; Pilgrim, 1996) of deformed area. However, the evaluation carried out by the authors showed that the largest tolerable deformation percentage varies with the location of the deformation. Only when the deformation is at the center of the surface can such values as 25 percent be achieved. However, in most circumstances, 16 percent is the upper limit, very close to the value of its theoretical break-down point - 14.3 percent.

To improve the tolerable percentage of deformed areas, data snooping techniques have been integrated with M-estimators. Experimental results show that the percentage is improved to a level of 22 percent from 16 percent. This integrated technique is also sensitive to the location of local deformation.

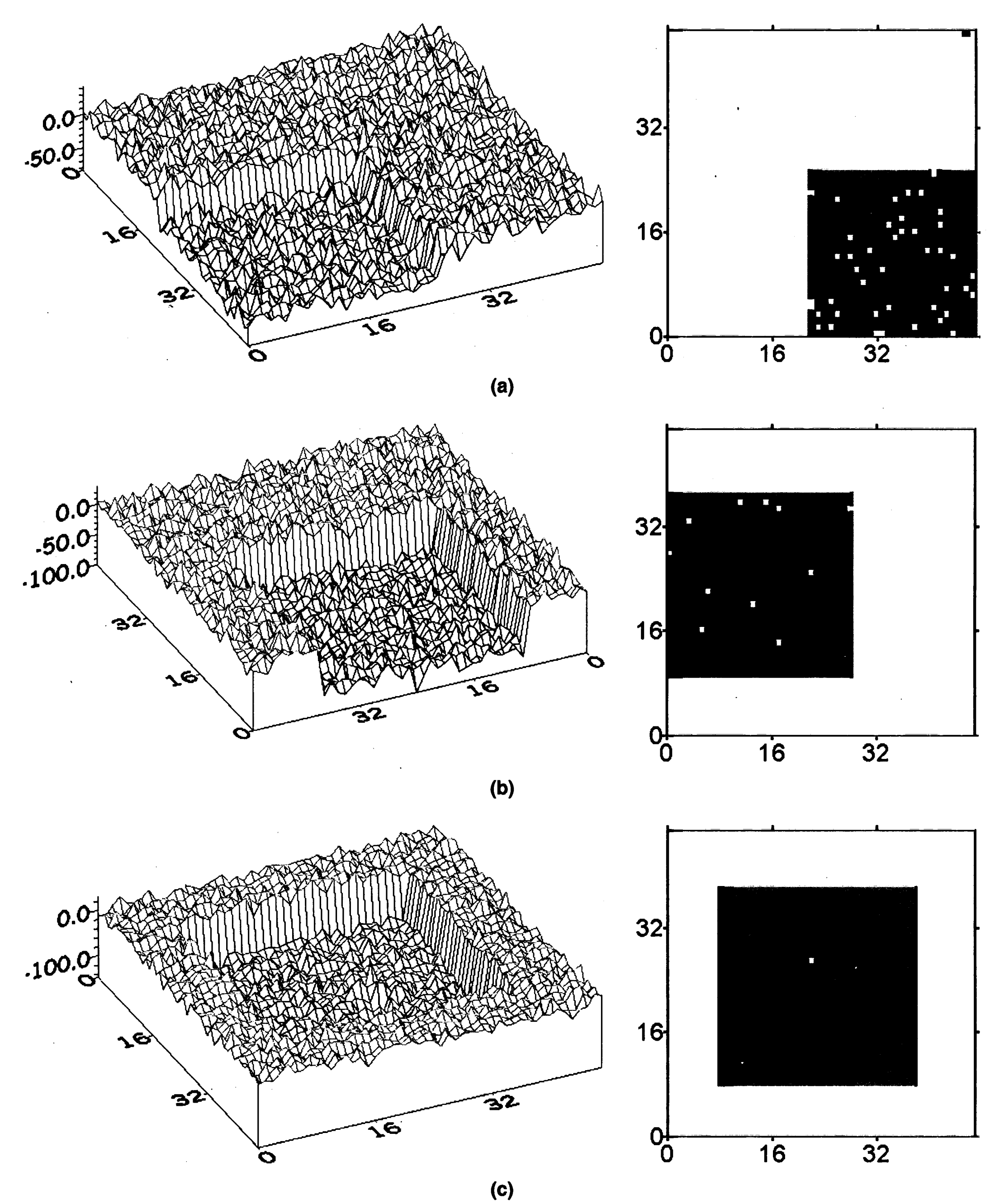

Figure 5. Performance of the LHD+LMS algorithm in detecting large-area small-magnitude deformation. (a) The deformation is located at the lower-right corner, occupies 30 percent of the entire area, and is of magnitude 5σ. (b) The deformation is located at the middle-left corner, occupies 36 percent of the entire area, and is of magnitude 7σ. (c) The deformation is located at the center, occupies 40 percent of the entire area, and is of magnitude 7σ.

The applicability of least-median-of-squares (LMS) has been carried out. The results show that LMS-estimators are able to tolerate local deformation up to 50 percent of the surfaces being matched. If the deformation percentage is less than 36 percent, local deformation with very small magnitude is also detectable. The LMS-estimator has another advantage over others, i.e., it is invariant with the location of local deformations. Therefore, it can be concluded that the LMS-estimator is superior to M-estimators for the detection of local deformation.

It was also found that the smallest detectable magnitude of deformation is related to the percentage of deformed areas. As the deformation percentage goes over 36 percent, the smallest detectable magnitude grows dramatically.

It needs to be noted here that, when the percentage of local deformation is near 50 percent of total surface area, then only deformations with large magnitudes can be detected by the LMS-estimator implemented by the authors. To make an improvement, the so-called S-estimators (Rousseeuw and Leroy, 1987) might be helpful; these have a relative efficiency much higher than that of the LMS-estimator but are still as

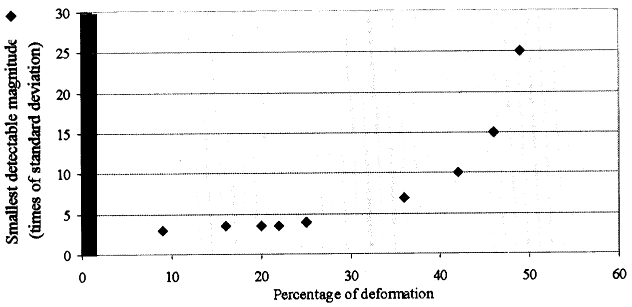

Figure 6. Percentage of deformation using the smallest detectable magnitude of local deformation by the LHD-LMS algorithm.

robust as the latter. However, a great increase in computation is expected if this method is used. Another estimator deserving attention is the MINPRA, developed by Stewart (1995). This estimator can tolerate more than 50 percent of outliers, with an assumption that the outliers are uniformly distributed. However, it is doubtful that such an assumption holds in the case of local deformation detection.

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