Expected Contributions of Dedicated Satellite Gravity Field Missions to Regional Geoid Determination with some Examples from Australia

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Abstract

Global models of the Earth's gravitational field can be divided among three primary classes: satellite-only, combined and tailored models. Some of the current deficiencies in some of these models will be outlined using examples over Australia. The new dedicated satellite gravity field missions (e.g., GRACE, CHAMP and GOCE) will make significant improvements to the long-wavelength, satellite-derived components of the Earth's gravitational field. They will also provide a homogeneous and near-complete global coverage of gravity field information. This paper briefly summarises the GRACE, CHAMP and GOCE mission concepts and expected error spectra based on previously published syntheses. Strategies will then be proposed for the use of these new data in regional gravimetric geoid computations based on data combination through either truncated spherical harmonic series or a deterministically modified Stokes formula.

1 Introduction

Since the 1960s, Earth-based tracking of geodetic satellites has been recognised as making the largest contribution to the determination of the long-wavelength components of the Earth's gravitational field on a global scale (e.g., Kaula, 1966). The derived global geopotential models (GGMs) are generally provided as coefficients of a truncated series expansion in terms of spherical harmonic basis functions, which can then be used to compute the external gravitational field. The maximum spatial resolution (half of the minimum wavelength) of the GGM (in km) at the Earth's surface is deduced by dividing 19,980-km by the maximum complete degree of the spherical harmonic expansion, while remembering the cosine effect of meridional convergence.

In their review, Lambeck and Coleman (1983) critique global geopotential modelling from 1958 to 1982. Importantly, this led to the provision of precision estimates for most subsequent GGMs. Authors such as Sneeuw (1994), Nerem *et al.* (1995) and Rapp (1997a) update and extend this review, and also outline some of the future prospects for global gravity field modelling. This paper attempts to partly build upon these reviews to include the prospects for gravity field determination with the advent of dedicated satellite gravity field missions, while acknowledging similar studies (e.g., Balmino *et al.*, 1999; Rummel *et al.*, 2002). However, this paper will focus more upon their contribution to regional gravimetric geoid modelling (cf. Tscherning *et al.*, 2002; Jekeli and Garcia, 2002), with particular reference to Australia.

2 Existing Global Geopotential Models

Current models of the Earth's gravitational field can be divided among three primary classes:

- a) satellite-only GGMs, derived from the tracking of artificial Earth satellites;
- b) *combined GGMs*, derived from a combination of a satellite-only model, terrestrial gravimetry, satellite altimeter-derived gravity data in marine areas, and (more recently) airborne gravimetry;
- c) *tailored GGMs*, derived from a refinement of existing (satellite or combined) GGMs using higher resolution gravity data that may have not necessarily have been used previously.

Parameters related to the Earth's external gravitational field are easily computed at any position from the spherical harmonic coefficients defining any GGM. This can be achieved using the

variant of Rapp's (1982) FORTRAN77 computer software, which is available from the US National Imagery and Mapping Agency (NIMA) (<u>http://164.214.2.59/GandG/wgs-84/egm96.html</u>). Several of the existing GGM coefficients (listed Tables 1 and 2) can be downloaded from the International Association of Geodesy's (IAG's) International Geoid Service (IGeS) (<u>http://www.iges.polimi.it/</u>), the Prof H-G. Wenzel's memorial website (<u>http://www.gik.uni-karlsruhe.de/~wenzel/geopmods.htm</u>), or the IAG's special working group on global gravity field modelling (<u>http://op.gfz-potsdam.de/iagwg/</u>). Alternatively, many authors provide their GGMs via anonymous FTP or directly from their websites.

2.1 Satellite-only global geopotential models

The estimation of GGM geopotential coefficients from Earth-based measurements of satellite orbital perturbations is described by, for example, Reigber (1989) and Lemoine *et al.* (1998). Though many of the recent satellite-only GGMs are available above spherical harmonic degree 50 (Table 1), the higher degree coefficients, say greater than 20 (e.g., Vaníček and Sjöberg, 1991) or 30 (e.g., Rummel *et al.*, 2002), are heavily contaminated by noise. This is due primarily to a combination of the:

- a) power-decay of the gravitational field with altitude (cf. Kaula's (1966) rule of thumb), coupled with the minimum satellite altitude being constrained by atmospheric drag;
- b) limited precision of the Earth-based range measurements to the satellites, primarily due to atmospheric refraction;
- c) inability to track complete satellite orbits (arcs), due to the limited coverage of Earth-based tracking stations, which is exacerbated for low-Earth orbiting satellites;
- d) imprecise modelling of non-gravitational and lunar, solar and planetary gravitational perturbations of the satellites' motion; and
- e) incomplete sampling of the global gravity field due to the limited number of satellite orbital inclinations available.

model	degree	citation	model	degree	citation
GEM-T2S	36*	Marsh et al. (1990)	GRIM4-S2	50	Schwintzer et al. (1992)
GEM-T3S	50	Lerch <i>et al.</i> (1994)	GRIM4-S3	50	Schwintzer et al. (1993)
JGM-1S	60	Nerem et al. (1994a and b)	GRIM4-S4	60*	Schwintzer et al. (1997)
JGM-2S	60	Nerem et al. (1994a and b)	EGM96S	70	Lemoine et al. (1998)
PGTF-4	50	Shum <i>et al.</i> (1990)	GRIM5-S1	99	Biancale et al. (2000)
GRIM4-S1	50	Schwintzer et al. (1991)			

Table 1. Some satellite-only global geopotential models published since 1990

* includes some additional (usually resonant) coefficients above this degree.

2.2 Combined global geopotential models

The maximum degree (and thus spatial resolution) of satellite-only GGMs can be increased to yield combined GGMs using terrestrial gravity and terrain data and satellite altimeter-derived gravity anomalies in marine areas. More recently, airborne gravity data have also been used in areas previously devoid of data, such as Greenland (e.g., Lemoine *et al.*, 1998). The satellite-only geopotential coefficients can also be adjusted as part of this data combination process according to the relative data weighting at the normal equation level. Rapp (1997a) describes the general philosophies behind the computation of combined GGMs, though the specific details for each model can be found in the references cited in Table 2 because the philosophies and techniques sometimes differ among groups.

It now appears to be widely acknowledged that EGM96 (Lemoine *et al.*, 1998) is generally the best of the current combined GGMs, whose spherical harmonic coefficients can be downloaded free-of-charge from <u>http://164.214.2.59/GandG/wgs-84/egm96.html</u>. Users that require the EGM96 geoid, as opposed to the quasi-geoid (Rapp, 1997b), can download 'correction' coefficients from the same website. This website also allows the user to interactively compute EGM96 geoid heights on-line, or to download an EGM96 geoid interpolation program for Microsoft Windows.

model	degree	citation	model	degree	citation	
GEM-T2	36*	Marsh et al. (1990)	GRIM5-C1	120	Gruber <i>et al.</i> (2000)	
GEM-T3	50	Lerch <i>et al</i> (1994)	OSU89A	360	Rapp and Pavlis (1990)	
JGM-1	70	Nerem et al. (1994a and b)	OSU89B	360	Rapp and Pavlis (1990)	
JGM-2	70	Nerem et al. (1994a and b)	OSU91A	360	Rapp et al. (1991)	
JGM-3	70	Tapley <i>et al.</i> (1996)	OGE12	360	Gruber and Bosch (1992)	
PGTF-4A	50	Shum <i>et al.</i> (1990)	GFZ93A	360	Gruber <i>et al.</i> (1993)	
TEG-2	54	Tapley <i>et al.</i> (1991)	GFZ93B	360	Gruber et al. (1993)	
TEG-2B	54	Tapley <i>et al.</i> (1991)	GFZ95A	360	Gruber et al. (1996)	
TEG-3	70	Tapley <i>et al.</i> (1997)	GFZ96	359	Gruber et al. (1997)	
GRIM4-C1	50	Reigber et al. (1993)	GFZ97	359	Gruber et al. (1997)	
GRIM4-C2	50	Reigber et al. (1993)	EGM96	360	Lemoine et al. (1998)	
GRIM4-C3	60	Schwintzer et al. (1993)	GAO-98	360	Demianov et al. (2000)	
GRIM4-C4	72	Schwintzer et al. (1997)	PGM2000A	360	Pavlis <i>et al.</i> (2000)	

Table 2. Some combined global geopotential models published since 1990

The primary limitations on the precision of combined GGMs are the spatial coverage and quality of the terrestrial (or airborne) gravity, terrain, and satellite altimeter data used, as well as the above-mentioned limitations on satellite-only GGMs. Clearly, in areas where no gravity data are available (due to restricted access or data confidentiality clauses), the satellite-only GGM cannot be improved, and may even be degraded as follows.

Data gaps can affect the combined GGM in other regions, because its determination is essentially based on the solution of a boundary-value problem (cf. Albertella *et al.*, 2001), where values over the entire boundary are required to determine that boundary (cf. Stokes's theorem). The effect of the data gaps is compounded by the use of global basis functions (i.e., spherical harmonics), which cannot preserve the multi-resolution character of the data (Blais and Provins, 2002).

There are also numerous other factors that affect the accuracy of combined GGMs, such as systematic errors in terrestrial gravity data (Heck, 1990). For instance, distortions in and offsets among different vertical geodetic datums (e.g., Featherstone, 1998), which are implicitly used to compute gravity anomalies, generate long-wavelength errors in terrestrial gravity anomalies. These cause low-frequency errors in the combined GGMs if not properly filtered from the combined solution (cf. Vanícek and Featherstone, 1998).

Therefore, combined GGMs, while generally offering an increased spatial resolution over satelliteonly GGMs, also vary in precision and accuracy from region to region.

2.3 Tailored global geopotential models

The so-called tailoring process can be used to refine satellite-only or combined GGMs, where the existing spherical harmonic coefficients are adjusted, and often extended to higher degrees, using (terrestrial, marine or airborne) gravity data that may not necessarily have been used before. This is normally achieved using integral formulas to derive corrections to the existing geopotential coefficients, as opposed to the combination at the normal equation level that is generally used to construct the combined GGMs.

Tailored GGMs can be developed either globally (e.g., Wenzel, 1998) or over a particular region of interest (e.g., Weber and Zommorrodian, 1988). Importantly, the regionally tailored GGMs *only* apply over the area in which the tailoring was applied, because spurious effects occur in areas where no data are available (e.g., Kearsley and Forsberg, 1990). This is analogous with the data coverage-related effects on combined GGMs (described above).

The tailored geopotential coefficients can be used with the computer software described earlier, after modifications to account for the higher degree and order terms. However, the computation of very high degree and order associated Legendre functions is both time consuming and becomes numerically unstable. Therefore, the efficient and stable routines proposed by Holmes and Featherstone (2002) are recommended in preference.

Wenzel (1998) computed the GPM98A, GPM98B and GPM98C globally tailored GGMs to spherical harmonic degree and order 1800. These geopotential coefficients can be downloaded from <u>http://www.gik.uni-karlsruhe.de/~wenzel/geopmods.htm</u>. They are based on the degree-20 expansion of EGM96 and a near global, 5-arc-minute by 5-arc-minute grid of terrestrial gravity anomalies. However, in areas where no terrestrial gravity data were available to Wenzel, such as Australia, the GPM98 models do not perform well, which will be demonstrated next.

3 Australian Examples of Errors in Existing Global Geopotential Models

Lambeck and Coleman (1983) make an extremely important point concerning all GGMs: "...the various models are not as good as they are said to be. If they were, the differences between them should not be so great as they are...". In this author's opinion, this statement is largely still true today. The following gives two examples of where this allegation is supported over Australia for the EGM96 and GPM98 GGMs.

In addition, the error estimate for any GGM is global and thus not necessarily representative of its performance in a particular region. As such, users of any GGM should perform their own accuracy and precision verifications, such as comparing GGM-derived gravity field quantities with local data (cf. Kirby et al., 1998). However, such comparisons are less informative in the low frequencies because of the well-known deficiencies in vertical geodetic datums (for comparisons with GPS-levelling data) and long-wavelength errors in terrestrial gravity data.

3.1 The EGM96 combined GGM

A potential problem with EGM96 has only recently been discovered over Australia, which is due to the use of two different digital elevation models (DEMs) during its construction. The JGP95E DEM (Lemoine *et al.*, 1998) was used in EGM96. To the west of the 140°E meridian, JGP95E is based on the TerrainBase DEM (Row *et al.*, 1995). To the east of the 140°E meridian, JGP95E is based on a DEM constructed from NIMA's topographic map holdings (Lemoine *et al.*, 1998). Figure 1 (from Hilton *et al.*, submitted) maps the differences between the JGP95E and TerrainBase DEMs, which clearly shows the disparity between them along the 140°E meridian.



Figure 1. Differences between the JGP95E and TerrainBase DEMs over Australia [linear projection; units in meters]

Since EGM96 was computed, the Australian Surveying and Land Information Group (AUSLIG, now the National Mapping Division of Geoscience Australia) has published two versions of the GEODATA 9 arc-second DEM of Australia (<u>http://www.auslig.gov.au/products/digidat/dem.htm</u>).

As well as offering a higher spatial resolution, these DEMs use additional data from Australia that were not used in JGP95E.

Version 1 of the GEODATA DEM was used to compute gravimetric terrain corrections over Australia, though the spatial resolution had to be decreased to 27 arc-seconds to avoid errors that were then attributed to instabilities in the terrain correction algorithm used (Kirby and Featherstone, 1999). This DEM and the 27 arc-second terrain corrections, as well as EGM96, were used in the computation of the AUSGeoid98 regional geoid model of Australia (Featherstone *et al.*, 2001). It has since been discovered (Kirby and Featherstone, 2001) that the numerical instabilities in the above terrain corrections were actually caused by gross errors in the version 1 GEODATA DEM. These have now been removed from version 2, and Kirby and Featherstone (in press) have computed a new grid of Australian gravimetric terrain corrections at the full 9 arc-second resolution.

Importantly, version 2 of the GEODATA 9 arc-second DEM of Australia, or subsequent releases, should be used in future combined GGMs and regional geoid models of Australia. Until then, the high degree and order terms in EGM96 should be treated with some caution over Australia.

3.2 The GPM98 tailored GGM

As stated earlier, the GPM98 models do not include terrestrial gravity data over Australia, which was due to data confidentiality clauses (Wenzel, 1998 pers. comm.). As expected from the work of Kearsley and Forsberg (1990), GPM98 performs poorly over Australia. This is demonstrated from a comparison of GPM98-implied quasi-geoid heights with a nation-wide set of 141 discrete heights derived from the algebraic difference between co-located Global Positioning System (GPS) ellipsoidal heights and spirit-levelled heights on the Australian Height Datum (AHD). From the results in Table 3, the GPM98 models, though they offer a higher spatial resolution, should not be used over Australia.

Model	Degree	Max (m)	Min (m)	Mean (m)	RMS (m)
EGM96	360	1.041	-0.857	0.071	0.388
GPM98A	1800	2.406	-1.509	0.250	0.698
GPM98B	1800	2.443	-1.516	0.232	0.705

Table 3. The fit of EGM96, GPM98A and GPM98B to 141 GPS-AHD heights in Australia (from Wenzel, 1998, personal communication)

4 Dedicated Satellite Gravity Field Missions: Concepts and Missions

The current (i.e., CHAMP and GRACE) and planned (i.e., GOCE) dedicated satellite gravity field missions will undoubtedly make a significant improvement to our knowledge of the long- and medium-wavelength (>200-km) components of the near-global gravity field. Current published expectations (summarised later) estimate that spherical harmonic degree coefficients less than ~200 (corresponding to spatial resolutions of greater than ~100-km) will be improved by approximately over one order of magnitude over existing GGMs (Tables 1 and 2).

At the broadest conceptual level, dedicated satellite gravity field missions observe (either directly or indirectly) the Earth's external gravitational gradients. This is essentially through differential measurements between two or more points, thus largely eliminating correlated errors. This can take two approaches (e.g., Rummel, 1979; Balmino *et al.*, 1999; Rummel *et al.*, 2002):

- a) satellite-to-satellite tracking (SST; e.g., Wolff, 1969; Douglas *et al.*, 1980; Sjöberg, 1982; Kaula, 1983; MacArthur and Posner, 1985; Wagner, 1987; Jekeli, 1999, 2002; Ilk, 2002);
- b) a dedicated gravity gradiometer instrument on board a satellite (e.g., Rummel and Colombo, 1985; Rummel, 1986; Petrovskaya, 1996; Petrovskaya and Zielinski, 1997, 2000).

The SST methods can use either low-low inter-satellite tracking (II-SST), where two low-Earth orbiting satellites track one another (e.g., Wagner, 1983; 1987; Cheng, 2002), or high-low inter-satellite tracking (hI-SST), where high-Earth orbiting satellites (notably GPS) track the low-Earth orbiting satellite(s) (e.g., Schrama, 1991; Visser and van den IJssel, 2000).

The satellite(s) being tracked should be in as-low-as-possible orbits, with the proof masses isolated, as-best-as-possible, from the perturbing effects of atmospheric drag. Generally, the SST methods are better at resolving the very low-frequency components of the global gravity field, whereas the low-Earth orbiting gradiometers are better at resolving the medium-frequency gravity field (shown later). Therefore, the logical approach is to use a combination (e.g., GOCE; described later).

Dedicated satellite gravimetry is heralded (correctly in this author's belief) as introducing a new era in gravity field determination (cf. Ilk, 2000). Various such missions have been proposed for over two decades (e.g., Kaula, 1983; Colombo, 1989), such as GRAVSAT (e.g., Piscine, 1982; Wagner, 1983), STAGE (e.g., Jekeli and Upadhyay, 1990), Aristotles (e.g., Visser *et al.*, 1994) and STEP (e.g., Albertella *et al.*, 1995a; Petrovskaya, 1997). However, it is only now that dedicated satellite gravity field missions have been or will be launched.

These missions will allow the computation of a whole new generation of GGMs, which will supersede the existing GGMs (Tables 1 and 2) and remedy most of their deficiencies (summarised earlier). Therefore, they will form an important basis for improved regional geoid modelling based on the remove-compute-restore technique and its many variants. The three dedicated gravity field missions that will be summarised in this paper are CHAMP, GRACE and GOCE, though other missions have been proposed, such as SAGE (Sansò *et al.*, 2001).

4.1 CHAMP

CHAMP (CHAllenging Mini-satellite Payload) is a German-led satellite mission to determine, among other Earth parameters, the static and time-varying components of the global gravity field (e.g., Reigber *et al.*, 1999 and 2000; <u>http://op.gfz-potsdam.de/champ/</u>). Here, the low-Earth orbiting (~454-km altitude) CHAMP satellite is tracked using the high-Earth orbiting (~20,200-km altitude) GPS satellites (Figure 2), relative to a network of ground stations, principally the stations of the International GPS Service network (<u>http://igscb.jpl.nasa.gov/</u>).



Figure 2. The CHAMP concept of satellite-to-satellite tracking in the high-low mode (from Rummel et al., 2002)

The benefit of using high-low SST for CHAMP is that the low-Earth orbit satellite will 'see' many GPS satellites and with a good constellation geometry for an entire orbit, while also being low enough to sense the higher frequencies in the Earth's gravity field. Three orthogonal accelerometers, oriented using star cameras, on board the CHAMP satellite are used to estimate the non-gravitational perturbations of its low-Earth orbit (e.g., Schwintzer *et al.*, 2000).

The CHAMP satellite was launched on 15 July 2000 and the mission is scheduled to run for approximately five years. It was placed in a near-circular orbit at an inclination of $\sim 87^{\circ}$ to the equatorial plane. This allows for a near-global coverage of observations with data gaps only at the poles. The CHAMP mission should allow determination of the near-global gravity field at a spatial resolution of ~ 650 -km. The accuracy of the derived GGM is expected to be more than one order of magnitude better than current satellite-only GGMs (Table 1), in the low frequencies (shown later).

4.2 GRACE

GRACE (Gravity Recovery And Climate Experiment) is a joint USA-German mission that follows on from CHAMP (e.g., Kim *et al.*, 2002; Jekeli, 2001; <u>http://essp.gsfc.nasa.gov/grace/;</u> <u>http://www.csr.utexas.edu/grace/</u>). The mission consists of two identical CHAMP-type satellites following one other in nearly the same orbit (~480-km altitude) separated by a distance of ~170-270-km; the so-called tandem formation. The low-satellite to low-satellite inter-tracking is measured using microwave links (e.g., Jekeli, 2000), coupled with high-satellite to low-satellite GPS-based SST tracking of both satellites (Figure 2).



Figure 3. The GRACE concept of satellite-to-satellite tracking in the low-low mode combined with satellite-to-satellite tracking in the high-low mode (from Rummel et al., 2002).

The five-year GRACE mission was launched on 17 March 2002 supposedly as a follow-on to CHAMP, though there will be an overlap period of approximately two years. The GRACE mission will improve upon the CHAMP determination of the global gravity field at low frequencies and will also increase the spatial resolution (shown later). The improvement in the low frequencies is because of the redundancy offered by the use of two low-Earth orbiting satellites, coupled with high-low SST. It will also allow time variations in the Earth's gravity field to be mapped approximately every 30 days.

4.3 GOCE

GOCE (Global Ocean Circulation Experiment) is a European Space Agency-led mission primarily to determine the global gravity field (e.g., <u>http://www.esa.int/export/esaLP/goce.html</u>; Rummel *et*

al., 2000, 2002; Klees *et al.*, 2000a, b, 2002; Sneeuw *et al.*, 2000; van Lonkhuyzen *et al.*, 2002; Visser *et al.*, 2002). The GOCE mission will use a low-Earth (~260-km altitude) orbiting satellite in a nearly Sun-synchronous orbit at 96.5 degrees inclination, together with high-satellite to low-satellite GPS/GLONASS-based tracking (Figure 4). Importantly, the GOCE satellite will house a dedicated three-axis electrostatic gravity gradiometer.



Figure 4. The COCE concept of satellite gravity gradiometry combined with high-low satellite tracking (from Rummel et al., 2002).

The GOCE satellite mission is due to be launched towards the end of 2005, and has an expected mission duration of \sim 20 months. This will allow determination of the stationary global gravity field at a spatial resolution of \sim 100-km, though there will be data gaps at the poles (e.g., Rudolph *et al.*, 2002). Based on the published literature, the GOCE mission is attracting a great deal of attention, with numerous simulations being conducted (summarised next).

5 Syntheses and Data Accuracy Expectations

It is important to acknowledge that this very early review-type paper of dedicated satellite gravity missions is unavoidably speculative. Firstly, GGMs derived from the launched GRACE and CHAMP missions have not yet been published widely, and GGMs obviously cannot yet be derived from GOCE.

However, numerous research groups around the world have conducted simulations to estimate the likely improvements that will be made to GGMs by these missions (e.g., Albertella *et al.*, 1995b, 2000; Arabelos and Tscherning, 1990, 1995, 1998; Balmino and Perosanz, 1995; Belikov *et al.*, 1995; Bettadpur *et al.*, 1992; Cheng, 2002; Cui and Lelgemann, 2000; Ditmar and Klees, 2002; Ilk, 2002; Jekeli, 1999, 2000, 2001; Kim *et al.*, 2002; Klees *et al.*, 2000a, b; Mackensie and Moore, 1997; Müller and Obendorfer, 1999; Obendorfer *et al.*, 2000; Obendorfer and Müller, 2000; Sneeuw and Ilk, 1997; Sneeuw *et al.*, 2001, 2002; Vermeer, 1991; Visser *et al.*, 2001). Many of the above-cited studies also propose numerous alternative theories for the determination of GGMs from satellite data, including its combination with terrestrial and airborne gravity data.

Importantly, each group uses different philosophies and computational approaches, but the relative merits of each will not be discussed here. Instead, only what appear to be the most representative examples will be used.

5.1 Computation of geoid heights and gravity anomalies

The above-mentioned dedicated satellite gravity field missions can be used individually or combined to create GGMs to describe the Earth's gravity field. Naturally, these are classified as satellite-only GGMs. The geoid height above the reference (normal) ellipsoid can be computed from the spherical harmonic coefficients to degree L using

$$N = \frac{GM}{r\gamma} \sum_{n=2}^{L} \sum_{m=0}^{n} (\delta \overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda) \overline{P}_{nm} (\cos \theta)$$
(1)

The GGM-implied gravity anomaly (at the Earth's surface) is computed to degree L using

$$\Delta g = \frac{GM}{r^2} \sum_{n=2}^{L} (n-1) \sum_{m=0}^{n} (\delta \overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda) \overline{P}_{nm} (\cos \theta)$$
(2)

where, in equations (1) and (2), *GM* is the geocentric gravitational constant, γ is normal gravity on the surface of the reference ellipsoid, (r,θ,λ) are the geocentric spherical polar coordinates of the computation point, \overline{P}_{nm} are the fully normalised associated Legendre functions for degree *n* and order *m*, and $\delta \overline{C}_{nm}$ and \overline{S}_{nm} are the fully normalised spherical harmonic coefficients that have been reduced by the even zonal harmonics of the reference ellipsoid. As stated, Holmes and Featherstone (2002) give efficient algorithms for the computation of fully normalised associated Legendre functions.

5.2 Simulated global geoid error spectra



Figure 5. Postulated cumulative geoid errors of dedicated satellite gravity field missions in relation to EGM96 (i.e., the best currently available GGMs)

The expected [global] precision of the geoid heights can be estimated from the error degree variances of the geopotential coefficients, which in turn are derived from the standard deviations estimated for each coefficient. The error degree variance of the geoid heights is

$$\sigma_N = \left(\frac{GM}{r\gamma}\right)^2 \sum_{m=0}^n (\sigma_C^2 + \sigma_S^2) \tag{3}$$

where σ_c and σ_s are the standard deviations of the geopotential coefficients. These geoid error degree variances can be cumulated (assuming [incorrectly] that no correlations occur among the coefficients) to give the total geoid height error to a particular resolution (Figure 5).

Figure 5 (author unknown) shows the cumulated *global* geoid error degree variances (cf. equation 3) that can be expected from the CHAMP, GRACE and GOCE missions in relation to EGM96; the best of the combined GGMs. The error degree variances in Figure 5 broadly correspond with the simulated error degree variances supplied by Visser (2002, pers. comm.). From the logarithmic vertical scale in Figure 5, it is clear that the dedicated gravity field missions perform much better than EGM96 in the low and medium frequencies, while acknowledging the potential contamination in the long-wavelengths of EGM96 by terrestrial gravity data.

It is also evident from Figure 5 that the different missions perform differently in different parts of the gravity field spectrum. This is entirely as expected because of the different mission concepts and parameters (described earlier). Therefore, a 'combined' satellite-only GGM that takes into account the relative weights of each satellite mission will probably produce an optimal GGM.

Figure 6 (from Rummel *et al.*, 2000) shows the cumulated [simulated] GOCE-derived geoid errors as a function of spherical co-latitude for varying maximum spherical harmonic degrees of expansion (and thus spatial resolution). The larger errors in towards the poles are due to the non-polar orbit of GOCE; the so-called polar gap problem (e.g., Sneeuw and van Gelderen, 1997; Rudolph *et al.*, 2002; Han *et al.*, 2001). These data gaps will need to be completed using terrestrial or airborne gravimetry (e.g., Bouman and Koop, 2001).



Figure 6. Cumulative geoid height errors for different degrees (L) as a function of spherical co-latitude

6 Two Suggested Data Combination Strategies for Regional Geoid Determination

It has been known for a long time that regional gravimetric geoid models are deficient in the long and medium wavelengths, which has often been attributed solely to errors in the GGM. However, Vaníček and Featherstone (1998) show that the use of an unmodified [spherical] Stokes kernel allows the un-attenuated propagation of terrestrial gravity data errors into the regional geoid solution. Therefore, given the well-known deficiencies in terrestrial gravity data (cf. Heck, 1990), it is plausible that these regional gravimetric geoid models have been contaminated by the terrestrial gravity data and not necessarily only by the GGM.

It is likely that the new GGMs will further reveal the low-frequency errors in terrestrial gravity data. Indeed, these new data will probably be so superior (cf. Figure 5) that the low- and medium-frequency terrestrial gravity data should be ignored altogether in many parts of the world. For example, regional gravimetric geoid computations in Australia indicate that the Australian terrestrial gravity data contain long-wavelength errors that contaminate the regional geoid solution (e.g., Forsberg, 1988; Featherstone *et al.*, 2001). However, distortions in the Australian Height Datum (cf. Featherstone, 1998) are also a plausible explanation.

Nevertheless, terrestrial gravity data can still add high-frequency gravity field information that cannot be sensed by the dedicated satellite gravity field missions (not even the low-orbiting gradiometers), and thus will remain valuable for regional geoid computations. Therefore, an appropriate data combination must be sought under the new 'conditions' set by the improved satellite-derived data (cf. Kusche *et al.*, 2002). Two strategies will be proposed here, as follows.

6.1 Truncated spherical harmonic series approach

Terrestrial gravity anomalies can be converted to a spherical harmonic series using analysis, giving what will be called a terrestrial-only GGM (e.g., Pavlis, 1998). These geopotential coefficients can be combined with the GGMs derived from dedicated satellite gravity field missions using a Weiner-type filter or similar technique (cf. van Gelderen and Koop, 1997) or at the normal equation level as for a combined GGM. However, the principal restriction to this approach is that the error degree variances for the terrestrial gravity data are generally unreliable, coupled with the use of global basis functions (both described earlier).

Instead, the new satellite-derived GGM coefficients can be used to simply replace the low degree coefficients of the terrestrial-only GGM. This is justified because (1) the new GGMs will give a homogeneous global coverage that the previous satellite-only GGMs could not (described earlier), and (2) it will completely filter all the terrestrial gravity data errors. These errors can then be mapped simply by evaluating equation 2 for the differences between the terrestrial-only coefficients and the new GGM-derived coefficients.

The GGM-implied geoid can be computed from this 'combined/tailored' model using equation 1. The quotation marks are used here because this is strictly neither a combined GGM (because the data combination has not been achieved at the normal equation level) nor a tailored model (because integral formulas have not been used). Instead, it uses a simple truncation and reassembly of two spherical harmonic series. Therefore, this approach, while arguably sound, remains somewhat speculative and empirical tests should be performed to assess its viability in relation to the alternative approaches.

6.2 Deterministically modified Stokes's formula approach

The well-accepted technique for regional geoid determination is through the combination of a GGM with terrestrial gravity data via an adaptation or modification of Stokes's formula. Here, the GGM is used to generate gravity anomalies (equation 2) that are subtracted from the terrestrial gravity anomalies, a regional residual geoid model computed from numerical integration of Stokes's formula, and these regional residual geoid undulations added to the GGM-implied geoid undulations (equation 1). Based on the errors in the low-frequency terrestrial gravity data in relation to the dedicated satellite gravity field missions, it becomes sensible to select a modification to Stokes's kernel that is the most powerful high-pass filter of terrestrial gravity data errors.

Vanícek and Featherstone (1998) demonstrate that the spheroidal Stokes kernel (equation 4) is the most effective high-pass filter of the kernels that they tested. Depending upon one's viewpoint,

this is equivalent to the Wong and Gore (1969) kernel modification, where the low frequencies are subtracted from Stokes's kernel to give the spheroidal kernel

$$S^{M1+1}(\psi) = S(\psi) - \sum_{n=2}^{M} \frac{2n+1}{n-1} P_n(\cos\psi)$$
(4)

where $S(\psi)$ is the spherical Stokes kernel and $P_n(\cos\psi)$ is the n-th degree Legendre polynomial. The degree *M* of spheroidal kernel (cf. Wong and Gore modification) must be less than the degree of GGM used; otherwise additional terms will arise (cf. Evans and Featherstone, 2000). In this scheme, the kernel defined by equation (4) simply replaces the spherical Stokes kernel usually used in most regional gravimetric geoid determinations.

However, the degree of the spheroidal kernel (M) in equation (4) can be a somewhat arbitrary choice. Previous studies (e.g., Vanícek and Sjöberg, 1991; Featherstone *et al.*, 2001) have chosen M = 20 since this is the point beyond which [current] satellite-only GGMs appear to become unreliable based on their error degree variance spectra (notwithstanding resonant terms). Assuming that the values in Figure 5 are representative of what can be expected from the dedicated satellite gravity field missions, appropriate degrees of spheroidal modification could be $M=\sim40$ for CHAMP, $M=\sim120$ for GRACE and even $M=\sim230$ for COGE. Of course a GGM derived from a combination of these (and other) satellite missions will yield different values, as will different cut-off criteria.

However, recall that the geoid error degree variance of a GGM is, by definition, global (equation 3). As such, the choice of M should more realistically be a function of position. This applies especially to the current GGMs, where the appropriate value should be chosen empirically (e.g., by fits to GPS-levelling data; e.g., Featherstone *et al.*, 2001). However, the GGMs derived from the dedicated satellite gravity field missions will have a good homogeneous precision (excepting the polar gaps). Therefore, the choice of the value of M can be chosen more justifiably from the new GGM's global error degree variance spectra.

7 Concluding Summary

This paper has reviewed some of the problems with existing GGMs, with case-study examples from Australia, followed by a brief summary of the mission concepts and parameters of the GRACE, CHAMP and GOCE dedicated satellite gravity field missions. Given that these missions will deliver an unprecedented level of precision in the determination of the low- and medium-frequency, near-global gravity field (excepting the polar gaps), they will further demonstrate the long- and medium-wavelength deficiencies in terrestrial gravity data.

Therefore, two proposals have been made for the combination of the new GGMs derived from these dedicated satellite gravity field data with terrestrial gravity data for regional geoid determination. The first is more speculative, where the satellite-only and terrestrial-only GGMs are truncated and merged with one another. The second is based on more well-established methods, where the data combination is achieved using a deterministically modified Stokes integral to filter the errors from the terrestrial gravity data

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