Evaluation of Geo-potential Models EGM96, WDM94 and GPM98CR in Hong Kong and Shenzhen

Zhicai LUO and Yongqi CHEN

Department of Land Surveying & Geo-Informatics, The Hong Kong Polytechnic University
Hung Hom, Kowloon, Hong Kong
e-mail: lszcluo@polyu.edu.hk, lsyqchen@polyu.edu.hk

Abstract

To select the best geo-potential model for the determination of Hong Kong and Shenzhen local geoids, three models - EGM96, WDM94 and GPM98CR - were tested in the region using GPS/leveling data and gravity measurements. The data include GPS/leveling observations at 43 stations, 640 gravity measurements covering the whole territory of Hong Kong, and 62 high precision GPS/leveling observations and 4870 gravity observations in Shenzhen. The results show that the geoid undulations from these geo-potential models contain systematic errors both in Hong Kong and in Shenzhen. After the biases are removed using GPS/leveling data, the STD of geoid undulation differences for EGM96, WDM94 and GPM98CR are 0.039m, 0.037m and 0.030m respectively in Hong Kong, and 0.052m, 0.032m and 0.095m in Shenzhen. While the STD of gravity anomaly differences for EGM96, WDM94 and GPM98CR are 14.136mGal, 13.559mGal and 15.130mGal respectively in Hong Kong, and 8.815mGal, 8.545mGal and 11.567mGal in Shenzhen, it is clear that the WDM94 model is the best of the three.

1 Introduction

The earth's gravity field, generally expressed as a set of spherical or ellipsoidal harmonic coefficients (called the geo-potential model), represents important and fundamental information for the geosciences, such as geophysics, geodynamics, oceanography, and surveying and mapping. Therefore, much effort has been made to develop precise geo-potential models, such as the OSU series, i.e., OSU89a, OSU89b and OSU91a (Rapp and Pavlis, 1990; Rapp et al., 1991), the GFZ series, i.e., GFZ93a, GFZ93b and GFZ95a (Gruber and Anzenhofer, 1993; Gruber et al., 1995), the NASA/GSFC and NIMA joint geo-potential model EGM96 (Pavlis, 1996), etc.

Many quantities of the earth's gravity field, such as geoid undulation, gravity anomaly, and deflection of the vertical, can be derived from a geo-potential model. A precise geo-potential model has been widely used as a reference in the determination of a local geoid using the remove-restore technique. For this purpose, the model, which best fits the gravity field quantities in the area of interest, is preferred. Taking into account the results of Yang (1998), we evaluate in this paper the following three models using the data available in Hong Kong and in Shenzhen: GPM98CR with degree and order 720 (Wenzel, 1998), EGM96 with degree and order 360 (Pavlis, 1996), and WDM94 with degree and order 360 (Ning et al., 1994). The data used include 43 precise GPS/leveling observations and 640 gravity measurements covering the whole territory of Hong Kong (about 1000 km^2), and 62 high precision GPS/leveling observations and 4870 gravity measurements in the Shenzhen Special Economic Zone of China (about 2000 km^2). Because the two regions have different height systems, although they are neighbors, the tests are conducted separately.

2 Test and evaluation of geoid undulation

2.1 GPS/Leveling data

There are 43 high quality GPS stations covering the whole territory of Hong Kong, whose heights above the Hong Kong Principal Datum were determined with precise geometric or trigonometric

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leveling. The accuracy specification of precise leveling is $4\sqrt{k}$, where k is the length of leveling line (in kilometers) and therefore the estimated accuracy of heights is around 2 cm. Shenzhen has 62 high quality GPS stations, surveyed in 2001. Their heights above China's 1956 Yellow Sea Height Datum were determined using precise geometric leveling. The estimated accuracy of the heights of these GPS stations is better than 2cm.

2.2 The method of evaluation

Let h and H denote the GPS-derived ellipsoidal height and the leveled height at a certain point, respectively. The geoid undulation at the point can be calculated by

$$N_{obs} = h - H \tag{1}$$

The value N_{obs} so obtained here is called the observed geoid undulation. The geoid undulation can also be computed from a geo-potential model (Heiskanen and Moritz, 1967):

$$N_{GM} = \frac{GM}{r \cdot \gamma} \sum_{n=2}^{n\max} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} (\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda)\, \overline{P}_{nm}(\sin\varphi) \tag{2}$$

where GM is the geocentric gravitational constant, γ is the normal gravity of the computation point, a is the long radius of the reference ellipsoid, *nmax* is the maximum degree of geopotential model, φ, λ and r are the geocentric latitude, longitude and radial distance of the computation point, $\overline{C}_{nm}$ and $\overline{S}_{nm}$ are the fully normalized coefficients of the anomalous potential, and $\overline{P}_{nm}(\sin\varphi)$ is the fully normalized associated Legendre function.

From formulae (1) and (2) the differences between the observed geoid undulations and those computed from a geo-potential model at the same position are:

$$\Delta N = N_{obs} - N_{GM} \tag{3}$$

The differences after removing the possible biases represent the goodness of fit of a geo-potential model to the observed quantities in the area.

2.3 The results

Figure 1 shows the distributions of the differences ΔN for three geo-potential models with respect to normal distribution curves for Hong Kong, and Figure 2 shows those for Shenzhen. Table 1 gives the relevant statistics. It can be seen that there exist systematic biases in the differences. The systematic biases may come from a difference between the gravimetric geoid, and the leveling datum, and the long-wavelength systematic errors in N_{GM} . A least squares fitting was carried out with the following four-parameter transformation:

$$N_{obs}(x, y) = N_{GM}(x, y) + a_0 + a_1 x + a_2 y + a_3 xy \tag{4}$$

where x and y in km are oriented towards east and north respectively in the HK80 grid coordinates or the Shenzhen local grid coordinates, and a_0 , a_1 , a_2 and a_3 are the four unknown parameters to be determined. The tests on the significance of the estimated unknown transformation parameters are given in Table 2. The test is insignificant for the case of GPM98CR for Hong Kong. After deletion of some insignificant parameters, however, it becomes significant.

One can see from Table 1 that the STD values of fitting for EGM96, WDM94 and GPM98CR are 0.132m, 0.073m and 0.160m in Hong Kong, and 0.080m, 0.140m and 0.201m in Shenzhen respectively before the transformation. After the least squares fitting, the STD values change to 0.039m, 0.037m and 0.030m in Hong Kong and 0.052m, 0.032m and 0.095m in Shenzhen respectively. Tables 3 and 4 give the relative accuracy between two stations. Figures 3 and 4 clearly show the changes of relative accuracy of geoid undulations N_{GM} with respect to baseline length for all cases. Overall, WDM94 provides better results. This is due to the fact that the gravity measurements in China were used in the development of WDM94.

Table 1 Statistics of the geoid undulation differences ΔN in Hong Kong (HK) and in Shenzhen (SZ) (Unit : meter)

Geo-potential model		Max		Min		Mean		STD	
		HK	SZ	HK	SZ	HK	SZ	HK	SZ
EGM96	before	0.356	0.033	-0.119	-0.339	0.100	-0.137	±0.132	±0.080
	after	0.090	0.127	-0.105	-0.117	0.000	0.000	± 0.039	±0.052
WDM94	before	0.580	0.895	0.239	0.125	0.407	0.416	±0.073	±0.140
	after	0.080	0.076	-0.118	-0.077	0.000	0.000	± 0.037	±0.032
GPM98CR	before	0.284	0.308	-0.280	-0.518	0.038	-0.158	±0.160	±0.201
	after	0.082	0.227	-0.064	-0.216	0.000	0.000	± 0.030	±0.095

Note: 'before' denotes the values before the systematic bias is removed, and 'after' denotes the values after the systematic bias is removed.

Table 2 Tests on the significance of the 4-parameter transformation

Item	EGM96		WDM94		GPM98CR	
	HK	SZ	HK	SZ	HK	SZ
Average (m)	0.100	-0.137	0.407	0.416	0.038	-0.158
STD (m)	±0.132	±0.080	±0.073	±0.140	±0.160	±0.201
Statistic value	4.98	-13.426	36.46	23.389	1.55	-6.178
Significant or not	YES	YES	YES	YES	NO	YES
Remarks	$t_{\alpha/2}(42) = 2.02$, $t_{\alpha/2}(61) = 2.00$, $\alpha = 0.05$					

Table 3 Relative differences of the geoid undulation in Hong Kong

Baseline Length (km)	Relative differences (ppm)							
	Leveling		EGM96		WDM94		GPM98CR	
	$4\sqrt{k}$	$8\sqrt{k}$	before	after	before	after	before	after
0 - 5	2.53	5.06	12.40	8.94	8.38	8.69	12.25	8.61
5 - 10	1.46	2.92	8.96	5.12	5.98	5.02	10.08	4.48
10 - 15	1.13	2.26	8.28	4.09	4.84	3.85	9.58	3.44
15 - 20	0.96	1.91	8.27	3.11	4.57	2.94	9.98	2.47
20 - 25	0.84	1.69	8.43	2.82	4.30	2.64	9.84	2.06
25 - 30	0.76	1.53	7.71	1.93	3.92	1.67	9.41	1.42
30 - 35	0.70	1.40	7.71	1.63	3.76	1.60	9.08	1.26
35 - 40	0.65	1.31	6.80	1.40	4.01	1.50	8.31	1.09
40 - 45	0.61	1.23	6.72	1.27	3.98	1.33	7.98	1.04
45 - 50	0.58	1.16	5.74	1.57	4.46	1.62	7.84	1.13
>50	0.55	1.10	3.59	1.88	2.66	1.84	7.66	1.07

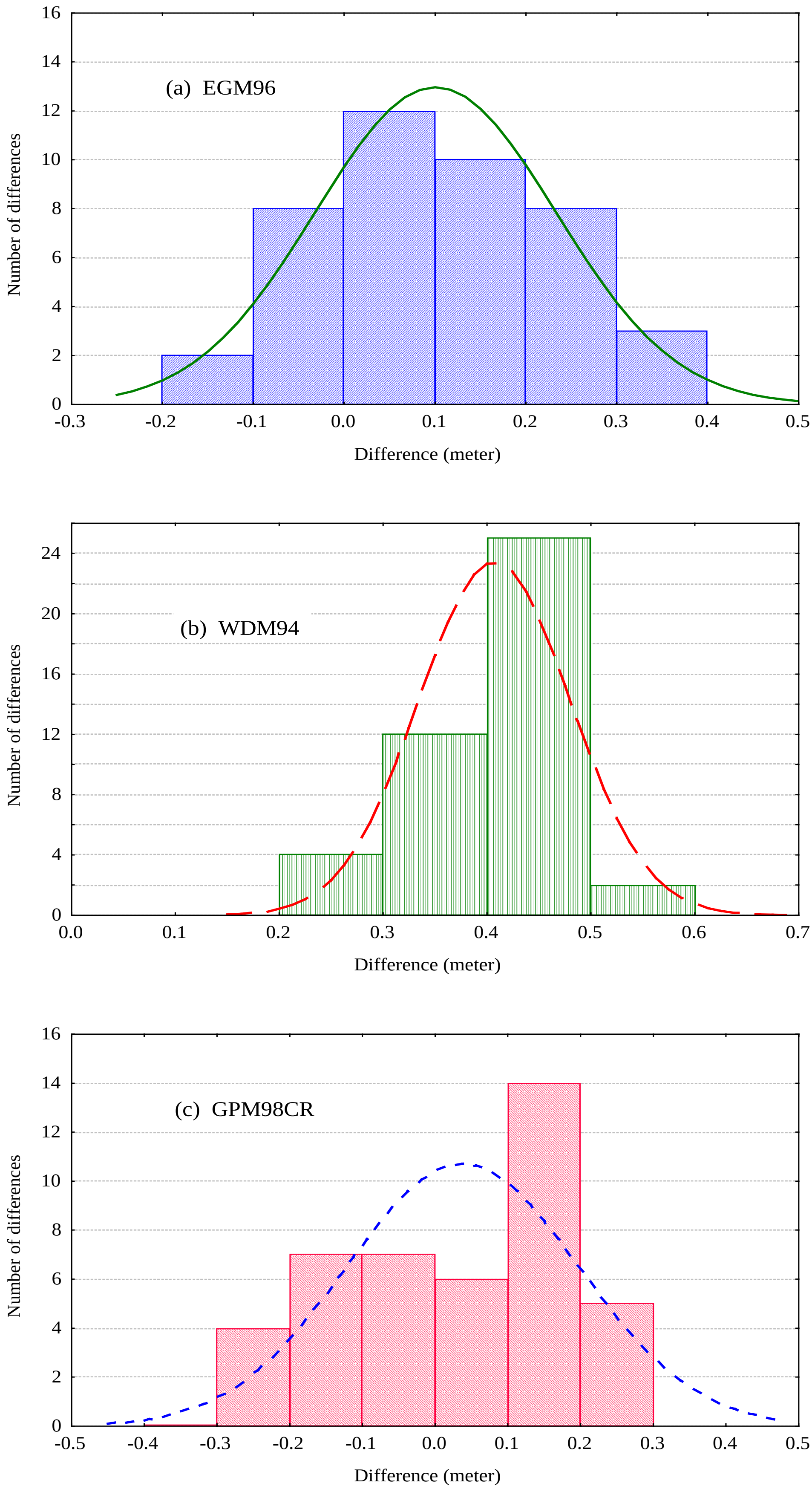

Figure 1 Histograms of the geoid undulation differences ΔN in Hong Kong

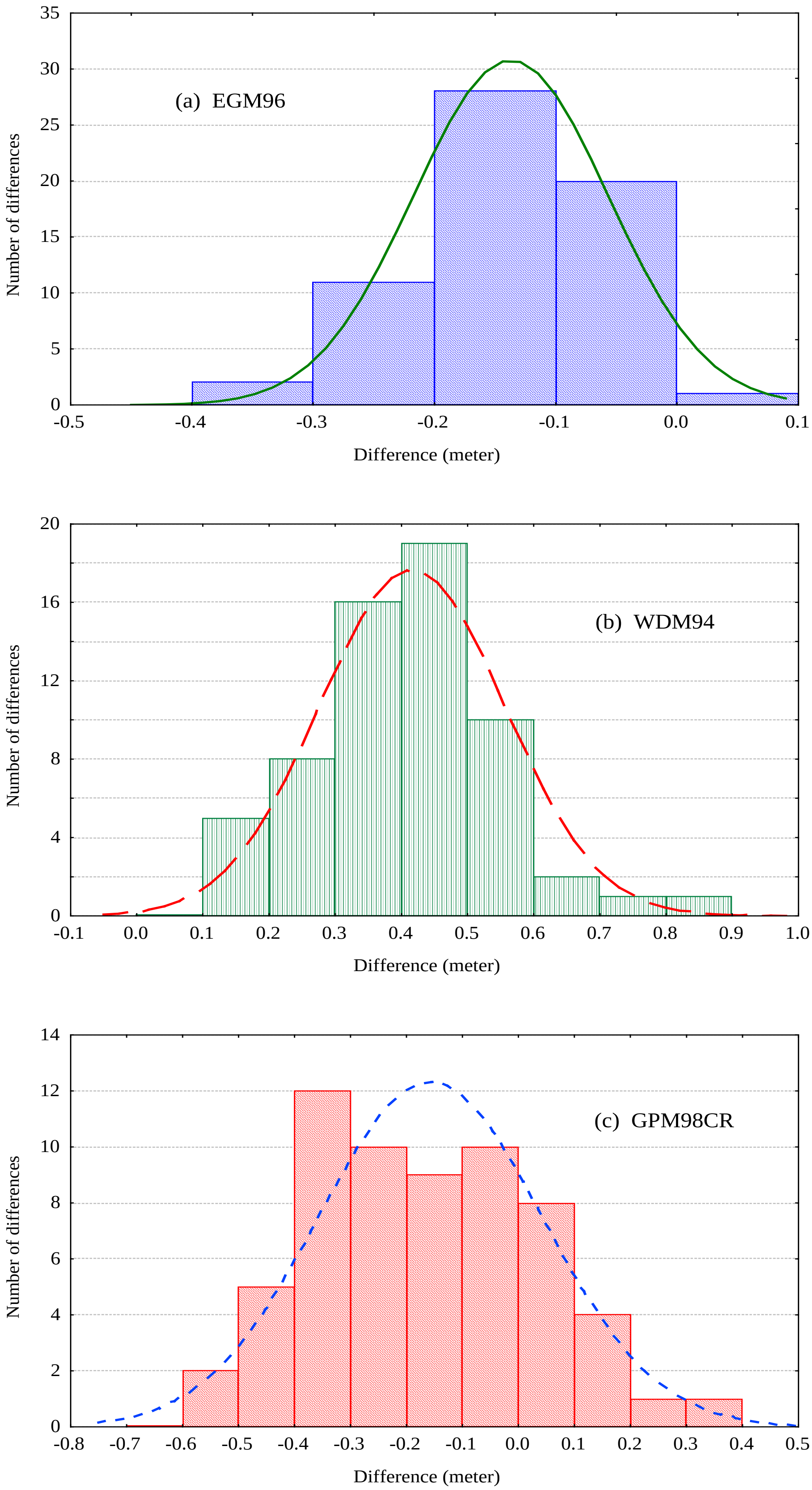

Figure 2 Histograms of the geoid undulation differences ΔN in Shenzhen

Table 4 Relative differences of the geoid undulation in Shenzhen

Baseline Length (km)	Relative differences (ppm)					
	EGM96		WDM94		GPM98CR	
	before	after	before	after	before	after
0 - 5	8.35	7.59	10.21	6.65	11.31	9.89
5 - 10	6.03	5.19	8.00	4.44	8.36	6.59
10 - 15	5.05	4.30	6.70	3.33	7.77	6.04
15 - 20	4.68	3.46	6.84	2.63	7.71	5.28
20 - 25	4.19	3.10	6.23	2.16	7.53	4.77
25 - 30	4.11	2.57	6.36	1.68	7.46	4.24
30 - 35	3.81	2.34	5.95	1.48	7.64	3.97
35 - 40	3.38	2.02	5.61	1.34	7.41	3.65
40 - 45	3.36	1.80	5.58	1.06	7.28	3.31
45 - 50	2.80	1.60	4.91	0.90	7.31	3.17
>50	2.06	1.34	4.18	0.63	6.68	2.74

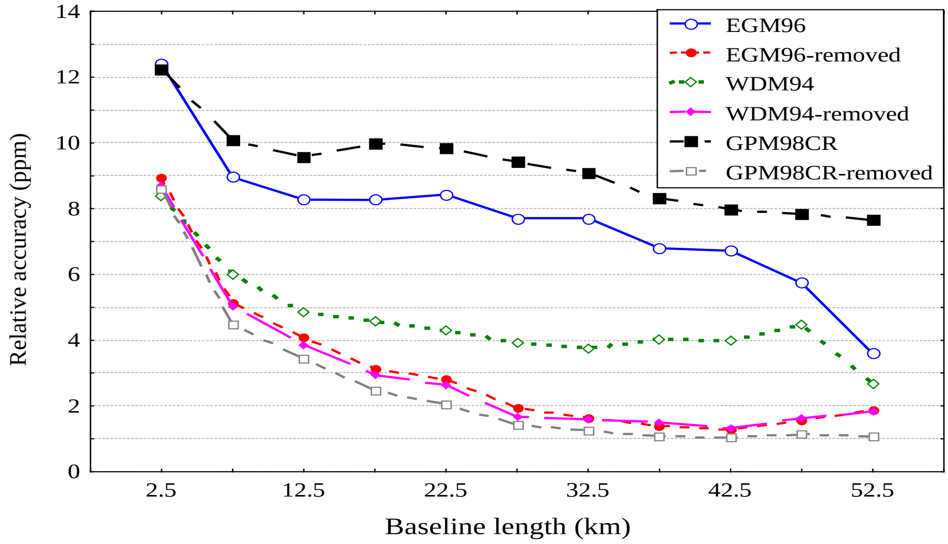

Figure 3 Relative differences of the geoid undulation in Hong Kong

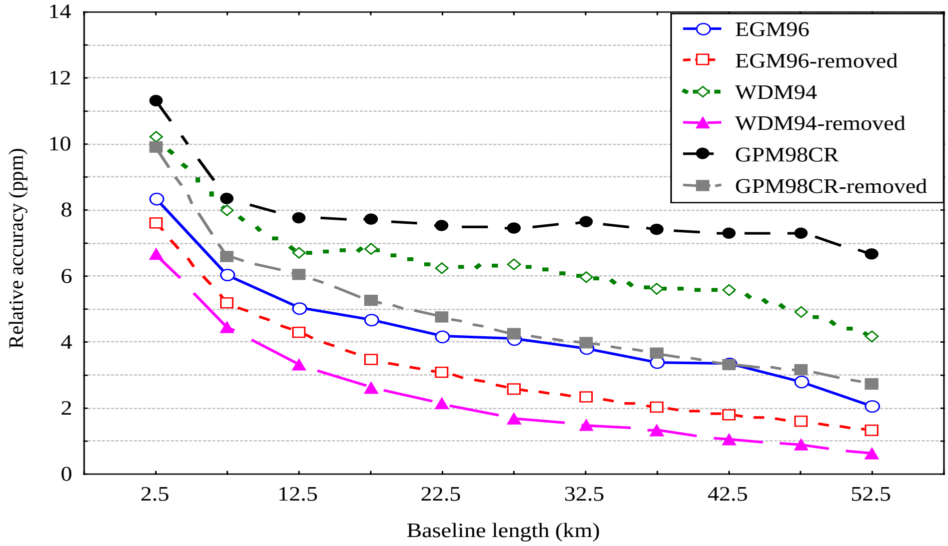

Figure 4 Relative differences of the geoid undulation in Shenzhen

3 Test and evaluation of gravity anomaly

In Hong Kong, 640 discrete gravity observations, with approximate station spacing of 2 kilometers on land and 2-4 kilometers on sea, have been collected using a Lacoste and Romberg model ‘G’ land gravity meter and a model ‘H/U’ seabed gravity meter by Electronic and Geophysical Services Ltd. (EGS) (EGS, 1988;1991). The local gravity base was connected to the International Gravity Standardization Net 1971 (IGSN 71) with accuracy of about 0.03 mGal (Evans, 1990). To determine the high precision Shenzhen geoid, 4870 discrete gravity observations (3608 points on land and 1262 points on sea) with 1 kilometer station spacing were collected in 2001 using the Lacoste & Romberg model ‘G’ and ‘D’ land gravimeter and the model ‘S’ sea gravimeter. The gravity observations were reduced onto the geoid (approximately the mean sea level) and the free-air gravity anomalies Δg_{obs} were obtained with WGS84 as a reference. The gravity anomalies so obtained are “observed quantities”. Furthermore, the gravity anomalies Δg_{GM} at the gravity stations can be computed from a geo-potential model (Heiskanen and Moritz, 1967):

$$\Delta g_{GM} = \frac{GM}{r^2}\sum_{n=2}^{n\max}\left(\frac{a}{r}\right)^n (n-1)\sum_{m=0}^{n}(\overline{C}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda)\,\overline{P}_{nm}(\sin\varphi) \qquad (5)$$

where all quantities have the same meanings as those in equation (2). The statistical information of Δg_{obs} and Δg_{GM} is listed in Table 5. The difference of the gravity anomalies between the observed and the computed from a geo-potential model can be obtained by

$$dg = \Delta g_{obs} - \Delta g_{GM} \qquad (6)$$

The differences contain random errors in Δg_{obs} and Δg_{GM}, systematic biases between Δg_{obs} and Δg_{GM}, long-wavelength systematic errors in Δg_{GM}, the high frequency part not included in Δg_{GM}, the theoretical approximations in the computation of Δg_{obs} and Δg_{GM}, etc. Figures 5 and 6 show the distribution of the gravity anomaly differences. From Table 6 we can see that the accuracies (standard deviation) of the gravity anomaly differences for geo-potential models EGM96, WDM94 and GPM98CR are 14.136mGal, 13.559mGal and 15.130mGal respectively in Hong Kong and 8.815mGal, 8.545mGal and 11.567mGal in Shenzhen. These results also indicate that WDM94 is slightly better than the other two geo-potential models.

Table 5 Statistics of the gravity anomalies Δg_{obs} and Δg_{GM} in Hong Kong and in Shenzhen (Unit: mGal)

Item	Max		Min		Mean		STD	
	HK	SZ	HK	SZ	HK	SZ	HK	SZ
EGM96	-0.332	0.553	-11.320	-8.630	-3.113	-3.426	±1.459	±2.530
WDM94	-3.863	-4.444	-12.621	-13.297	-7.540	-7.959	±1.974	±1.810
GPM98CR	7.456	2.117	-20.619	-25.491	-3.113	-5.371	±5.251	±6.899
Observed	48.674	59.844	-33.043	-32.800	-13.180	-11.541	±14.106	±8.832

Table 6 Statistics of the gravity anomaly differences dg in Hong Kong and in Shenzhen (Unit: mGal)

Geo-potential model	Max		Min		Mean		STD	
	HK	SZ	HK	SZ	HK	SZ	HK	SZ
EGM96	51.653	64.509	-29.729	-27.593	-10.067	-8.115	±14.136	±8.815
WDM94	57.842	68.619	-23.139	-21.298	-5.640	-3.582	±13.559	±8.545
GPM98CR	49.287	65.745	-33.995	-34.355	-10.068	-6.170	±15.130	±11.567

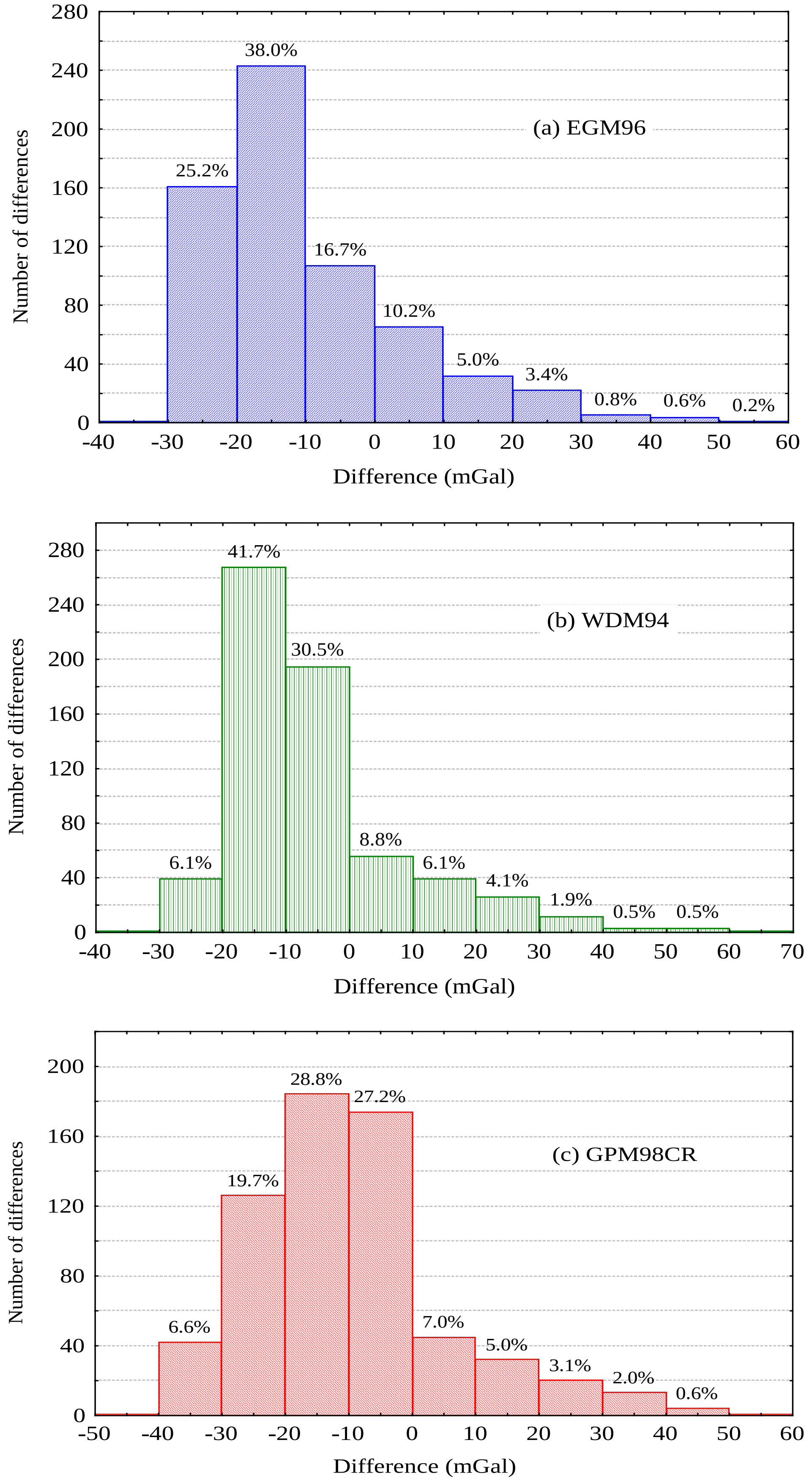

Figure 5 Histograms of the gravity anomaly differences dg in Hong Kong

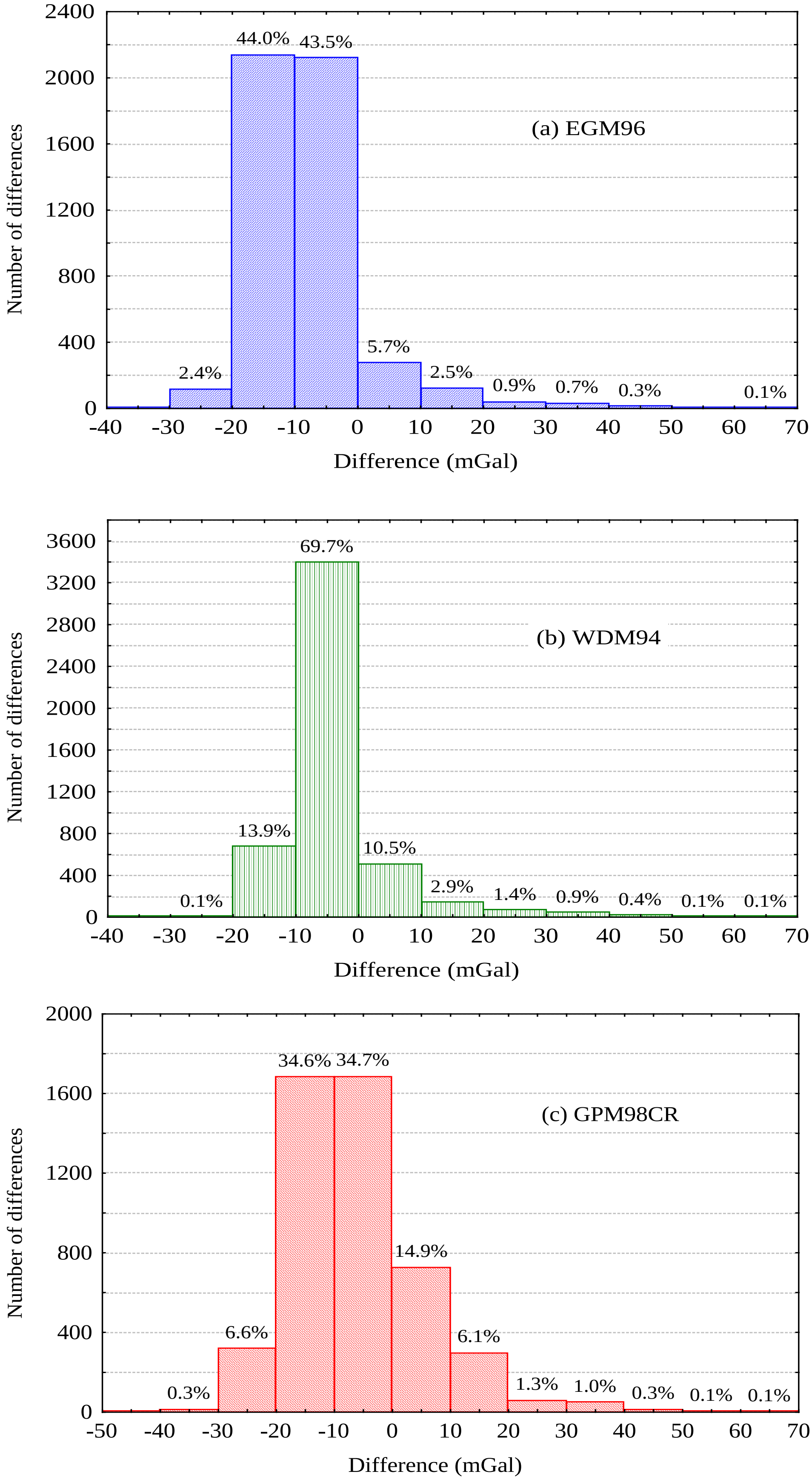

Figure 6 Histograms of the gravity anomaly differences dg **in Shenzhen**

4 Conclusion

This paper evaluates different geo-potential models in a regional environment (Hong Kong and Shenzhen). Three geo-potential models, EGM96, WDM94, and GPM98CR, were tested using GPS/leveling data and gravity measurements. The results indicate that WDM94 is the best of the three, though GPM98CR has higher resolution than the other two.

Acknowledgements

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